Stochastic Calculus and Finance
Übungen
Teil 3

Problem 6

Let $(X_n)_{n\geq 1}$ be a sequence of identically distributed independent random variables with the density function given by the formula

$$f(x)_{a,b} = \begin{cases} a, & if\ 0 \leq x \leq 1 \\ \exp(-bx), & if\ x > 1\,. \end{cases}$$

Define new stochastic sequences $Y_n = \sum_{i=1}^{n} X_i$ and $Z_n = \prod_{i=1}^{n} X_i$. Are there such positive parameters a, *b* that $(Y_n)_{n\geq 1}$ and/or $(Z_n)_{n\geq 1}$ are martingales with respect to filtration $F_n = \sigma\{X_1, \dots\dots X_n\}$?

Problem 7

Assume a polynomial form for the term structure of zero-coupon yields with the shape parameters:

$a)\ A_0 = 0{,}08, \quad A_1 = 0{,}02, \quad A_2 = -0{,}003, \quad A_3 = 0{,}0001$

$b)\ A_0 = 0{,}06, \quad A_1 = 0{,}01, \quad A_2 = -0{,}001, \quad A_3 = 0{,}0001$

The bond face value, annual coupon and maturity are *1000$*, *5%*, and *4* year. In both cases, find the price and the percentage change of the bond price, assuming the short rate increasing by 40 basic points, the slope decreasing by 10 basic points, and the changes of other parameters are zero.

Problem 8

In the framework of the Black-Scholes model of a (B, S) -market consider an investment portfolio π with the initial capital x. Estimate the asymptotic profitability of π:

$$\lim_{T\to\infty} sup \frac{1}{T} \ln E\,(X_T^{\pi}(x))^{\delta}, \qquad \delta \in\,]0.1].$$

Hint*. Use the Lyapunov's inequality.*