CDO Correlation Smile/Skew in One-Factor Copula Models: An Extension with Smoothly Truncated α-Stable Distributions

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Abstract

We propose a one-factor model for credit derivatives with smoothly truncated stable distributed factors which combines the parsimony of the copula model structure with the flexibility of stable distributions.

The one-factor copula model has become the market standard to price CDOs and tranched CDS index products. But the use of the normal distribution, as well as many alternatives as factor distributions, has lead to poorly reproduced market tranche spreads which resulted in the well-known correlation smiles. This short-coming is cured to a sufficient extent by our model.

1 Introduction

In the last decade, the market for credit derivatives has grown immensely.[1] This development has been accompanied by the emergence of various valuation techniques and models for credit risk. In addition to the attractiveness of credit derivatives for risk management, an important reason for the numerous publications is without doubt the fact that the recent years have been challenging for these financial products. For example, the extensive use of credit derivatives (in particular credit default swaps (CDS)) is conceived as one of the factors of the "correlation crisis" in 2005 and the "subprime mortgage crisis" in 2007.

Investors and insurance companies suffered huge losses, and the impact of these crises is still perceptible in almost all economies throughout the world.

In May 2005, the derivative models went through a very tough test triggered by a downgrading of Ford and General Motors in April and May of 2005, respectively. The sharp rise in idiosyncratic risk coincided with CDS spreads widening to record levels.

It is common knowledge that the simple basic models are not capable of reproducing market prices correctly. A common phenomenon encountered in this context is the so-called correlation smile occuring when a correlation coefficient is estimated for each tranche to match the respective market spreads. On the other hand, more advanced models quickly become too difficult handle. The more sophisticated a model is, the more likely it will be liable to overfitting and thus becoming too inflexible for changes. A multitude of parameters often face an insufficient data basis for estimation. For these reasons, many authors such as, for example, Collin-Dufresne et al. (2001), prefer simple models as given by the one-factor copula model.

Before we proceed into the model itself, we briefly introduce the concept of a particular financial instrument for credit risk, the synthetic credit default obligation (CDO). In brief, the synthetic CDO is a securitization of a pool of credit default swaps (CDS) related to certain reference entities or titles. Since the value of each CDS depends on the probability of default on the entity it is contingent on, the entire construct of the synthetic CDO will consequently be determined by the joint probability of default of the entirety of the titles.

[1] See, for example, Wang et al. (2006)

2 One-factor Copula Model

2.1 Valuation of Credit Derivatives

In contrast to the firm-value approach first conceived by Merton (1974) based on the value of the underlying entity issuing the bond which is modeled as a geometric Brownian motion and still developed further such as by Hull et al. (2009), the reduced-form or intensity-based model presented in Li (2000) and Duffie and Singleton (2003), for example, concentrates directly on the probability of default of the bond within a given period of time. We will follow the second approach.

The probability of default obtained from the homogenous Poisson process with intensity λ representing the distribution of the credit event of some defaultable zero-bond related to the exponentially distributed inter-arrival time τ between two successive jumps (i.e., credit events) is denoted as

$$F(t, \tau) = 1 - e^{-\lambda(T-t)} \tag{1}$$

that is the conditional probability of default within the next T units of time conditional on t units of time with no default. Consequently, the conditional survival probability of the next T units of time is given by

$$S(t, \tau) = 1 - F(t, \tau) = e^{-\lambda(T-t)} \tag{2}$$

The so-called credit triangle unique to the intensity-based model of the recovery rate, R, the CDS spread, $s_0^{CDS}(T)$, and the default intensity, λ obtained from the equality of the present values of premium and protection legs, i.e., the expected present values of payments of the respective counterparties, is given by

$$\lambda = \frac{1}{\delta} \cdot ln \left(\frac{s_0^{CDS}(T) \cdot \delta}{1 - R} + 1 \right). \tag{3}$$

where spread payments are made at the discrete dates t_k, $k = 1, \ldots, v$, with a constant time-lag of $\delta = (t_k - t_{k-1})$ between any two successive payment dates and the additional assumption that defaults can only happen at the spread payment dates. The proof of (3) can be found in the Appendix.

To relate the payments connected to tranche i with attachment and detachment points K_{i-1} and K_i to its market spreads, we introduce the single tranche (STCDO). Then, with the relative portfolio loss $L(t)$ the percentage loss of tranche i is given by

$$\Lambda_{\langle K_{i-1};K_i\rangle}(L(t)) = \frac{(max\,[0, L(t) - K_{i-1}] - max\,[0, L(t) - K_i])}{K_i - K_{i-1}} \tag{4}$$

from whence as a consequence of the equality of the present values of the premium and proctection leg we compute the tranche spread

$$s^{STCDO}_{\langle K_{i-1};K_i\rangle}(0,T) \approx \frac{\sum\limits_{k=1}^{v} B(0,t_k)\,[E_i(L(t_k)|\mathcal{F}_0) - E_i(L(t_{k-1})|\mathcal{F}_0)]}{\sum\limits_{k=1}^{v} \delta B(0,t_k)\,[1 - E_i(L(t_k)|\mathcal{F}_0)]} \tag{5}$$

with $B(0,t_k)$ denoting the zero bonds maturing at t_k. In the approximation (5), we used the notation $E_i(L(t_k)|\mathcal{F}_0)= \mathbb{E}(\Lambda_{\langle K_{i-1};K_i\rangle}(L(t_k))|\mathcal{F}_0)$. $\mathcal{F}_0$ is the information at time t_0.

2.2 Extensions of the One-Factor Model

The one-factor model from Vasicek (1987) builds on the concept the so-called Large Homogeneous Portfolio (LHP) model, a widely used market standard for the credit index families CDX and iTraxx as it is easy to understand and implement. It has been serving as the foundation for various extensions such as, for example, Kalemanova et al. (2005) or Hull and White (2004).

For the LHP, we will assume that the number of entities in the reference portfolio is very large. Each reference entity in the portfolio will have the same homogenized face value ω, correlation between any two entities given by ϱ is constant, the recovery rate is $R = 0.40$ for all entities, each reference entity will default with the same time-dependent probability p_t, and the default intensity λ is given to be constant at any time. We model the return of entity i at time t as

$$b_{i,t} = \sqrt{\varrho} \cdot Y_t + \sqrt{1-\varrho} \cdot \epsilon_{i,t}, \tag{6}$$

where the market factor Y and the idiosyncratic factor ϵ_i are independent standard normal random variables. The returns are thus standardized, i.e., we have $\mathbb{E}(b_{i,t}) = 0$ and variance $Var(b_{i,t}) = (\sqrt{\varrho_i})^2 + (\sqrt{1-\varrho_i})^2 = 1$.

Now, we consider some extensions to the Vasicek model with respect to heavy-talied distributions of the factors. The first alternative is the well-known Student's-t distribution, and the second is the truncated stable distribution.[2]

With Student's t distributed factors Y_t and $\epsilon_{i,t}$ with identical degrees of freedom, ν, the standardized return of firm i is now given by

$$b_{i,t} = \sqrt{\varrho} \cdot \sqrt{\frac{\nu - 2}{\nu}} Y_t + \sqrt{1 - \varrho} \cdot \sqrt{\frac{\nu - 2}{\nu}} \epsilon_{i,t},$$

The four parameter α-stable probability distributions (denoted either $S_\alpha(\sigma, \beta, \mu)$ or sometimes $S(\alpha, \sigma, \beta\, \mu))$ commend themselves for the use in asset return modeling because of the pleasant property of stability under summation and linear transformation.[3] Their main shortfall with respect to finite empirical data and most asset price models, however, is that they, in general, have no finite variance. Nonetheless, for example, Prange and Scherer (2009) used the α-stable distribution to model spreads of tranched CDS index products. They found that this distribution fit the spreads in all classes quite well.

Moreover, for most parameter values, the density function is unknown and has to be approximated. The most common approach is via the characteristic function. The method applied by the authors is based on the inverse Fourier approach by Chenyao et al. (1999) for values in the center of the distribution which provides an extension of the original method first conceived by DuMouchel (1971) who suggested to additionally use the series expansion developed by Bergström (1952) for values in the tails of the distribution where the inverse Fourier transform tends to fail to produce reliable results. We base our implementation on Menn and Rachev (2004a).

To cope with the non-existence of moments of all orders, we use a truncated version of the stable distribution - the so called smoothly truncted α-stable (STS) - first introduced by Menn and Rachev (2004b). Denoted by $s_\alpha^{[a,b]}(\sigma, \beta, \mu)$, the STS distributions combine α stable tails with a normal center between quantiles a and b to yield tail probabilities of arbitrary magnitude and finite moments of all orders.

[2]For a review on different distributional alternatives in the copula model, we recommend Wang et al. (2006).

[3]Stable distributions have been enjoying a wide field of applications involving changes of large magnitudes not only in finance. See, for example, Stuck (2000). For a thorough treatment of this distribution class, we recommend Samorodnitsky and Taqqu (1994).

In Papenbrock et al. (2009), for example, an analysis of credit derivative prices with truncated stable distributions was performed in the context of a structural model approach.

Furthermore, the class of STS distributions is closed under linear transformations such that we can convert any STS random variable into a standardized transform with zero mean and unit variance. The STS version of (6) uses independent identically standard STS distributed systematic market and idiosyncratic factors Y_t and $\epsilon_{i,t}$.
We have implemented an efficient Levenberg-Marquardt optimization routine for the determination of the truncation points a, b such that the STS distribution has zero mean and unit variance. We achieve an accuracy of 10^{-12} in an acceptable amount of computational time.

3 Model Setup and Data

The next step is to calculate the expected tranche loss

$$\mathbb{E}\left[\Lambda_{\langle K_{i-1};K_i\rangle}(L(t))\right] = \int_0^1 \Lambda_{\langle K_{i-1};K_i\rangle}(x[1-R])f^{(t)}(x)dx. \tag{7}$$

In the Appendix, we outline the theoretical framework to derive a closed-form expression for (7) for normally distributed factors. Due to the simplifying assumptions, it is obvious to use the closed-form expression from equation (13) in case of the normal distribution. The situation is not that simple in case of the Student's t- and STS distribution assumption. There exists no closed-form formula for the expected tranche loss. Instead, we have performed an integration method based on the Gaussian quadrature rule.
The required density in (7) is calculated from

$$\begin{aligned} f(x; p_t, \varrho) &= \frac{\partial F(x;p_t,\varrho)}{\partial x} \\ &= \varphi\left(\frac{\Phi^{-1}(p_t)-\sqrt{1-\varrho}\cdot\Phi^{-1}(x)}{\sqrt{\varrho}}\right)\cdot\frac{\sqrt{1-\varrho}}{\sqrt{\varrho}\cdot\varphi(\Phi^{-1}(x))}. \end{aligned} \tag{8}$$

where φ denotes the respective density of either the t-distribution or the STS distribution. The parameter (ϱ, θ) is obtained from calibration with respect to the equity tranche and minimizing the fit error expressed by the Euclidean norm.[4]

In order to perform the optimization procedure, we have implemented an extended grid search procedure for both distributions. For the degrees of freedom parameter ν, the search is performed on the set $(2, 250]$. For the STS distribution, we optimize over $\alpha \in [1.05, 1.99]$, $\beta \in [-0.6, 0.6]$, and $\sigma \in [0.15, 0.65]$.[5]

As market data, we use the CDX.NA.IG index – a portfolio of 125 equally weighted investment grade companies in the United States which is updated semiannually – and the corresponding tranche quotes with 5- and 10-year maturities.[6] We use weekly Tuesday market quotes.[7] Our data cover the period from June 22, 2004 until August 30, 2005 for the 5-year quotes, and the period between October 12, 2004 and August 30, 2005 for the 10-year quotes. Hence, both data sets include the time of the so called correlation crisis in May 2005.

4 Results

In Tables 1 through 4, we list the results from the estimation for the CDX.NA.IG 5Y separated into the overall period of observation (6-22-2004 until 8-30-2005), the perriod before (6-22-2004 until 4-26-2004), during (5-3-2004 until 6-28-2005), and after the crisis (7-5-2005 until 8-30-2005). In Tables 5 through 8, we repeat the same for the CDX.NA.IG 10Y. Each table is structured in the following manner. As indicated, in rows 2 through 4, we refer to the results for the $N(0, 1)$ distribution, while rows 4 through 6 and 8 through 10 list the results for the Student's t and STS distributions, respectively. For each distributon, the respective first two rows of column 2 through 6 refer to the ratios of the estimated spreads and the respective market spreads.

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[4]In fact, we minimize $rRMSE = \sqrt{\frac{1}{5}\sum_{k=2}^{5}\left(\frac{s_k - s_k^{Market}}{s_k^{Market}}\right)^2}$, where $k = (2, \ldots, 5)$ denotes all non-equity tranches such that the modeled and observed equity tranche spread always coincide.

[5]For ν, α, and σ, we used a grid size of 0.01, while for β it was 0.05.

[6]In contrast to CDS data on individual titles that are usually traded OTC, index prices are publicly available and supported by greater liquidity as stated, for example, in Gündüz et al. (2007).

[7]The market data we have analyzed consists of the CDX.NA.IG Index Series 3, CDX.NA.IG Index Series 4, and CDX.NA.IG Index Series 5.

Column 7 contains the maximum $rRMSE$ values for each distribution. As to the first and second rows for each distribution, in the first rows (i.e., rows 2, 5, and 8), we list the minimum ratios while in the respective second rows (i.e., rows 3, 6, and 9), we list the maximum ratios over the given period of time. Note that in the columns for the equity tranche, i.e. the columns labeled 3-7%, the entries are always 1 since the estimation is calibrated to exactly reproduce the spreads of this tranche. Also, for each distribution, we list the ranges for the parameter estimates.

We begin with the discussion of the results for the CDX.NA.IG 5Y. The overall results in Table 1 reveal that according to the $rRMSE$, the STS distribution outperforms the other competitors. We have $rRMSE = 0.12$ for the STS, whereas the values for the Student's t and standard normal copula are both greater than 1. With respect to the correlation ϱ estimates, we find that they are varying between roughly 15% and 25% for the normal- and t-copula while the bandwidth of the ρ estimates for the STS-copula is much narrower with values ranging from 23% to 29%. The STS-copula obviously tends to put more weight on the market factor in the composition of each entity's return. A greater correlation coefficient helps to level the generally more extreme outcomes of the two independent factors when STS distributed to yield returns for the entities that reflect a rather smooth market environment. By and large, we find that the STS-copula provides good or even excellent results for all except the first mezzanine tranche spreads which it tends to underestimate. We attribute this to the shortcoming of the stable component of the STS distribution to assign sufficient probability to small (negative) values necessary for scenarios with few defaults only.

Moreover, a glance at the graphics in Figures 1 and 2, where we display the compound correlation estimates of each tranche for the entire period, reveals quite a distinct correlation smile for both the normal and t copula. This is in contrast to the STS copula, where the correlation appears nearly constant across all tranches at any given time of observation, except for the period during the crisis. The latter is depicted in Figure 3 where we also see an almost constant correlation over time for the periods before and after the crisis.[8]

From the more detailed information provided by Tables 2 through 4, we see that, in general, all distributions are challanged by the downgrades. How-

[8]The compound correlation estimates are not tabulated in the contribution and can be obtained from the authors upon request.

ever, the STS proves superior. It is striking that for all three distributions, ρ is estimated lower during the crisis than before and after.
TABLE 2 ABOUT HERE !!!
TABLE 3 ABOUT HERE !!!
TABLE 4 ABOUT HERE !!!

In Tables 5 through 8, we list the estimation results for the CDX.NA.IG 10Y. The tables are structured in the same manner as for the CDX.NA.IG 5Y. The tables reveal that all three models generally yield poorer results. The results for the normal and the t copula, however, with $rRMSE$ of up to about 141% for both are much worse than for the STS with $rRMSE$ of 28%. In general, the STS provides the best results, though it has the tendency to underestimate the 3-7% spreads. This is in sharp contrast to the competitors that yield reasonable results for this tranche but overestimate in virtually all other tranches. Moreover, correlation is higher for all three models for this index than for the five year index. The bandwidth of the ϱ estimates for the STS copula is relatively narrow with values between 24% and 30% thus providing the market factor again with the heighest weights in the return dynamics (6).

Moreover, from a glance at Figures 4 through 6, where we display the compound correlation estimates of each tranche for the entire period, we notice a very remarkable skew accross the tranches for the normal and t copula.[9] In contrast, the STS correlation remains relatively flat except for the peak of the crisis where ϱ surges for the lower mezzanine tranches.

Next, we look at the different periods in detail. From Table 6 through 7, we see that during the crisis, the $rRMSE$ is much worse for the normal- and t-copula than before, while the $rRMSE$ indicates only slightly poorer fit for the STS-copula. Furthermore, Table 8 reveals that results have generally improved after the crisis compared to the prior period. However, the normal- and the t copula provide very imprecise spread estimates with the $rRMSE$ well above 1 for both. The results for the STS-copula, on the other hand, are extremely good. y-tails.

TABLE 5 ABOUT HERE !!!
TABLE 6 ABOUT HERE !!!

[9] The compound correlation estimates are not tabulated in this contribution and can be obtained from the authors upon request.

TABLE 7 ABOUT HERE !!!
TABLE 8 ABOUT HERE !!!

5 Conclusion

We proposed an extension to the standard one-factor copula model by using the STS distribution risk both for the market factor as well as the idiosyncratic risk and compared it to the Gauss and t copula using the tranche spreads of the CDX.NA.IG 5Y and CDX.NA.IG 10Y indices, respectively. Our observation covered the period from June 22, 2004 until August 30, 2005 thus including the bond crisis of may and June, 2005.

As a measure of goodness-of-fit, we introduced the relative root mean square error ($rRMSE$) which considers the deviation of the modeled spreads from the observed market spreads, for each tranche. We saw that the results for the STS model proved often excellent and genreally much better than the normal and Student's t alternatives before, during, and after the crisis. The only short-coming was that the STS copula model persistently underestimated the first mezzanine tranche spreads slightly.

The commonly found correlation smiles accross the tranches found in the normal and the t copula model disappeared almost completely for our model extension.

At the time, we conducted the analysis, we did not consider the present crisis of world-wide impact which began in 2007. It will definitely be a very interesting question to pose as to whether the results would show similar superior behavior for the STS extension if we repeat the computationt for the two indices we used between 2007 and now.

6 Appendix

6.1 Proof of (3)

$$
\begin{aligned}
s_0^{CDS}(T) &= \frac{\sum_{k=1}^{v} B(0,t_k)(1-R)\overbrace{\left(e^{-\lambda t_{k-1}} - e^{-\lambda t_k}\right)}^{F(0,t_{k-1})-F(0,t_k)}}{\delta \cdot \sum_{k=1}^{v} B(0,t_k) \cdot e^{-\lambda t_k}} \\
&= \frac{\sum_{k=1}^{v} B(0,t_k)(1-R) \cdot e^{-\lambda t_k} \left(e^{-\lambda(\overbrace{t_{k-1} - t_k}^{-\delta})} - 1 \right)}{\delta \cdot \sum_{k=1}^{v} B(0,t_k) \cdot e^{-\lambda t_k}} \\
&= \frac{(1-R)\left(e^{\lambda\delta} - 1\right) \cdot \sum_{k=1}^{v} B(0,t_k) \cdot e^{-\lambda t_k}}{\delta \cdot \sum_{k=1}^{v} B(0,t_k) \cdot e^{-\lambda t_k}} \\
\Leftrightarrow \lambda &= \frac{1}{\delta} \cdot ln\left(\frac{s_0^{CDS}(T) \cdot \delta}{1-R} + 1 \right)
\end{aligned}
\tag{9}
$$

□

6.2 Derivation of the expctd tranche loss formula for normal factors.

In this Appendix, we outline the theory leading to the closed form of the expected tranche loss formula based on the assumptions in section 2.2.

Equality of the default probabilities, p_t, implies identical default barriers at any time t, i.e., $c_{i,t} = c_t$ for all n entities.[10] Then, it follows that the

[10] The default barrier is defined as the threshold of the return for the firm to default, i.e., $p_t = P(b_{i,t} < c_t)$.

default probability conditional on $Y = y_t$ is the same for all reference entities

$$p(y_t) = \Phi\left(\frac{\Phi^{-1}(p_t) - \sqrt{\varrho}y_t}{\sqrt{1-\varrho}}\right). \tag{10}$$

Now, let us introduce $L_i(t) = \quad (\tau_i < t)$ to indicate whether entity i has defaulted by t. Then, the relative portfolio loss is

$$L(t) := \sum_{i=1}^{n} w_i(t) \cdot L_i(t) \tag{11}$$

where $w_i(t)$ is entity i's exposure at default (EAD) as a fraction of the overall portfolio exposure at default at time t and $L_i(t)$ is the indicator of entity i's default, which is one when i has defaulted by t (i.e., $\tau_i < t$) and zero otherwise. For the portfolio loss, we can deduct the convergence in probability

$$L(t) \xrightarrow[n\to\infty]{P} p(Y_t). \tag{12}$$

To see this, let us assume:
$\mathbb{E}(L(t) \,|\, Y_t) = p(Y_t)$ and $Var(L(t) \,|\, Y_t) = \sum_{i=1}^{n} (w_i)^2 \cdot p(Y_t) \cdot (1 - p(Y_t))$.

We want to show that $\forall\, \epsilon > 0 : \lim\limits_{n\to\infty} P(|L(t) - p(Y_t)| \geq \epsilon) = 0$:

From:

$$\begin{aligned} \mathbb{E}[(L(t) - \mathbb{E}(L(t) \,|\, Y_t))^2] &= \mathbb{E}[\mathbb{E}[(L(t) - \mathbb{E}(L(t) \,|\, Y_t))^2] \,|\, Y_t] \\ &= \mathbb{E}[Var(L(t) \,|\, Y_t)] \end{aligned}$$

and

$$\begin{aligned} Var(L(t) \,|\, Y_t) &= \sum_{i=1}^{n} (w_i)^2 \cdot p(Y_t) \cdot (1 - p(Y_t)) \\ &\leq \frac{1}{4} \cdot \sum_{i=1}^{n} (w_i)^2 \xrightarrow[n\to\infty]{} 0 \end{aligned}$$

follows:

$$\mathbb{E}[(L(t) - p(Y_t))^2] = \mathbb{E}[(L(t) - \mathbb{E}(L(t) \,|\, Y_t))^2] \xrightarrow[n\to\infty]{} 0.$$

From convergence in $\mathbb{L}^2$ follows convergence in probability as stated. Consequently, we can conclude convergence in distribution and, hence, obtain for the unconditional portfolio loss distribution

$$\begin{aligned} F(x; p_t, \varrho) &= P(p(Y_t) \leq x) \\ &= \Phi\left(\frac{\sqrt{1-\varrho}\cdot\Phi^{-1}(x) - \Phi^{-1}(p_t)}{\sqrt{\varrho}}\right). \end{aligned}$$

from whence follows the expected tranche loss as the closed-form expression[11]

$$\begin{aligned} \mathbb{E}\left[\Lambda_{\langle A;B\rangle}(L(t_j))\right] = \frac{(1-R)}{B-A}\Bigg[\Phi_2\left(-\Phi^{-1}\left(\frac{A}{1-R}\right), c(t_j); -\sqrt{1-\varrho}\right) - \ldots \\ \ldots - \Phi_2\left(-\Phi^{-1}\left(\frac{B}{1-R}\right), c(t_j); -\sqrt{1-\varrho}\right)\Bigg]. \end{aligned} \tag{13}$$

where, Φ_2 denotes the bivariate normal distribution.

[11] For a detailed presentation, we recommend O'Kane and Schloegl (2001).

7 Exhibits

	0-3%	3-7%	7-10%	10-15%	15-30%	rRMSE
N(0,1)	1	1.1679	0.8822	0.5449	0.0492	1.1400
		2.7288	2.3066	1.4400	0.6999	
	$\varrho \in [0.12, 0.25]$					
t	1	1.0407	0.8750	1.1205	0.5931	1.0400
		2.2380	2.3113	2.0926	0.9693	
	$\varrho \in [0.16, 0.26]$, $\nu \in [5.30, 33.00]$					
STS	1	0.8400	0.8860	1.0306	0.8528	0.1200
		1.0439	1.1075	1.1708	0.9863	
	$\varrho \in [0.23, 0.29]$, $\alpha \in [1.00, 1.45]$, $\beta \in [-0.09, 0.05]$					
(1)	(2)	(3)	(4)	(5)	(6)	(7)

Table 1: Overall results for CDX.NA.IG 5Y

	0-3%	3-7%	7-10%	10-15%	15-30%	rRMSE
N(0,1)	1	1.1679	0.8822	0.7897	0.2300	0.5400
		1.5860	1.5923	1.3840	0.6999	
	$\varrho \in [0.19, 0.25]$					
t	1	1.0407	0.8750	1.1205	0.8533	0.4500
		1.4152	1.5469	1.6369	0.9693	
	$\varrho \in [0.25, 0.26]$, $\nu \in [8.50, 33.00]$					
STS	1	0.8816	0.8860	1.0435	0.9040	0.1000
		1.0439	1.0591	1.1547	0.9863	
	$\varrho \in [0.25, 0.29]$, $\alpha \in [1.07, 1.45]$, $\beta \in [0.01, 0.05]$					
(1)	(2)	(3)	(4)	(5)	(6)	(7)

Table 2: Results for CDX.NA.IG 5Y before crisis

	0-3%	3-7%	7-10%	10-15%	15-30%	rRMSE
N(0,1)	1	1.5211	1.2202	0.5449	0.0492	1.0700
		2.5937	2.0929	1.0424	0.2222	
	$\varrho \in [0.12, 0.18]$					
t	1	1.3014	1.4513	1.4057	0.5931	0.9300
		2.1135	2.2695	1.7476	0.8920	
	$\varrho \in [0.16, 0.22]$, $\nu \in [5.30, 9.00]$					
STS	1	0.8400	0.9580	1.0306	0.8607	0.1200
		0.9361	1.0938	1.1392	0.9609	
	$\varrho \in [0.23, 0.26]$, $\alpha \in [1.02, 1.18]$, $\beta \in [-0.09, 0.01]$					
(1)	(2)	(3)	(4)	(5)	(6)	(7)

Table 3: Results for CDX.NA.IG 5Y during crisis

	0-3%	3-7%	7-10%	10-15%	15-30%	rRMSE
N(0,1)	1	2.0520	1.5972	0.7237	0.0800	1.1400
		2.7288	2.3066	1.4400	0.2012	
	$\varrho \in [0.15, 0.20]$					
t	1	1.6564	1.7293	1.5132	0.7898	1.0400
		2.2380	2.3113	2.0926	0.8920	
	$\varrho \in [0.21, 0.24]$, $\nu \in [5.70, 5.80]$					
STS	1	0.8838	0.9772	1.0592	0.8528	0.1200
		0.9408	1.1075	1.1708	0.9450	
	$\varrho \in [0.24, 0.26]$, $\alpha \in [1.00, 1.16]$, $\beta \in [-0.04, 0.04]$					
(1)	(2)	(3)	(4)	(5)	(6)	(7)

Table 4: Results for CDX.NA.IG 5Y after crisis

	0-3%	3-7%	7-10%	10-15%	15-30%	rRMSE
N(0,1)	1	1.0218 1.3569	1.2182 2.9134	1.2695 3.5824	0.7046 1.2679	1.4098
	$\varrho \in [0.18, 0.27]$					
t	1	1.0155 1.3451	1.1885 2.7430	1.2831 3.5909	0.9438 1.4627	1.4142
	$\varrho \in [0.18, 0.28]$, $\nu \in [9.90, > 200)$					
STS	1	0.6283 0.9961	0.7955 1.1394	1.0232 1.3349	0.7941 1.0239	0.2806
	$\varrho \in [0.24, 0.30]$, $\alpha \in [1.02, 1.55]$, $\beta \in [-0.50, -0.05]$					
(1)	(2)	(3)	(4)	(5)	(6)	(7)

Table 5: Overall results for CDX.NA.IG 10Y

	0-3%	3-7%	7-10%	10-15%	15-30%	rRMSE
N(0,1)	1	1.1354 1.3569	1.2182 1.8192	1.2695 1.9496	0.7046 1.1400	0.6302
	$\varrho \in [0.21, 0.27]$					
t	1	1.1041 1.3451	1.1885 1.7849	1.2831 1.9533	1.0080 1.1628	0.6313
	$\varrho \in [0.22, 0.28]$, $\nu \in [11.30, > 200)$					
STS	1	0.8336 0.9961	0.9450 1.0823	1.0232 1.1083	0.9627 1.0239	0.0935
	$\varrho \in [0.29, 0.30]$, $\alpha \in [1.05, 1.55]$, $\beta \in [-0.5, -0.05]$					
(1)	(2)	(3)	(4)	(5)	(6)	(7)

Table 6: Results for CDX.NA.IG 10Y before crisis

	0-3%	3-7%	7-10%	10-15%	15-30%	rRMSE
N(0.1)	1	1.0218 1.2874	1.6090 2.7543	1.8984 3.5824	0.8757 1.2679	1.4098
	$\varrho \in [0.18, 0.21]$					
t	1	1.0155 1.2790	1.6000 2.6986	1.9158 3.5909	0.9438 1.4627	1.4142
	$\varrho \in [0.18, 0.22]$, $\nu \in [17.40, > 200)$					
STS	1	0.6283 0.9349	0.7955 1.1394	1.0963 1.3349	0.7941 0.9897	0.2806
	$\varrho \in [0.24, 0.28]$, $\alpha \in [1.02, 1.08]$, $\beta \in [-0.11, -0.05]$					
(1)	(2)	(3)	(4)	(5)	(6)	(7)

Table 7: Results for CDX.NA.IG 10Y during crisis

	0-3%	3-7%	7-10%	10-15%	15-30%	rRMSE
N(0,1)	1	1.1287 1.3021	2.1833 2.9134	2.0006 2.5460	0.7842 0.9841	1.2405
	$\varrho \in [0.19, 0.21]$					
t	1	1.0931 1.2106	2.1255 2.7430	2.0322 2.6455	0.9911 1.4403	1.2231
	$\varrho \in [0.20, 0.27]$, $\nu \in [7.50, 64.00])$					
STS	1	0.7074 0.8067	1.0008 1.0901	1.0504 1.1197	0.9535 1.0111	0.1566
	$\varrho \in [0.26, 0.30]$, $\alpha \in [1.02, 1.08]$, $\beta \in [-0.15, -0.05]$					
(1)	(2)	(3)	(4)	(5)	(6)	(7)

Table 8: Results for CDX.NA.IG 10Y after crisis

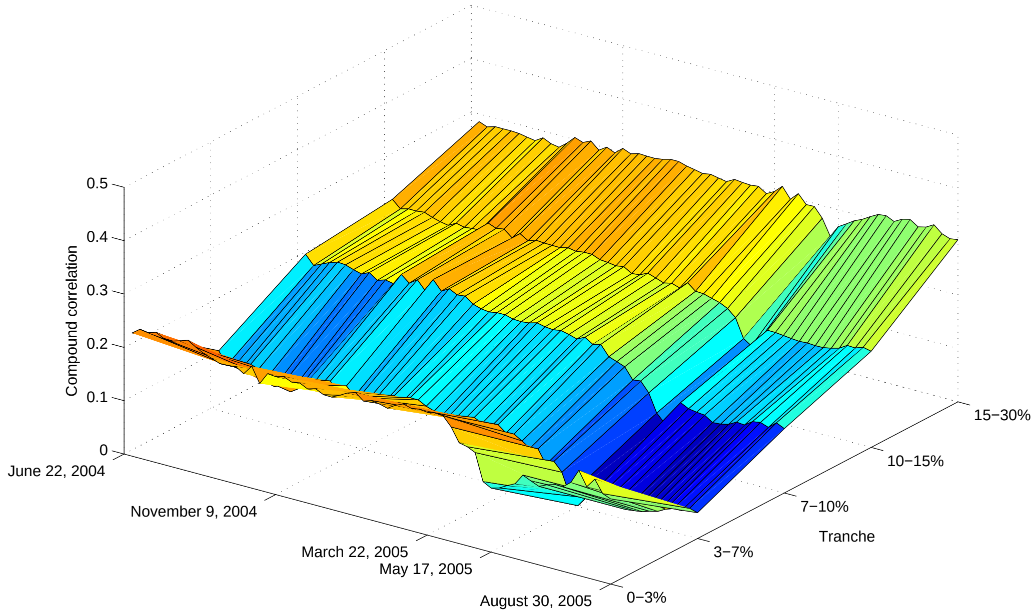

Figure 1: CDX.NA.IG 5Y: Compound correlation surface - Gaussian Copula.

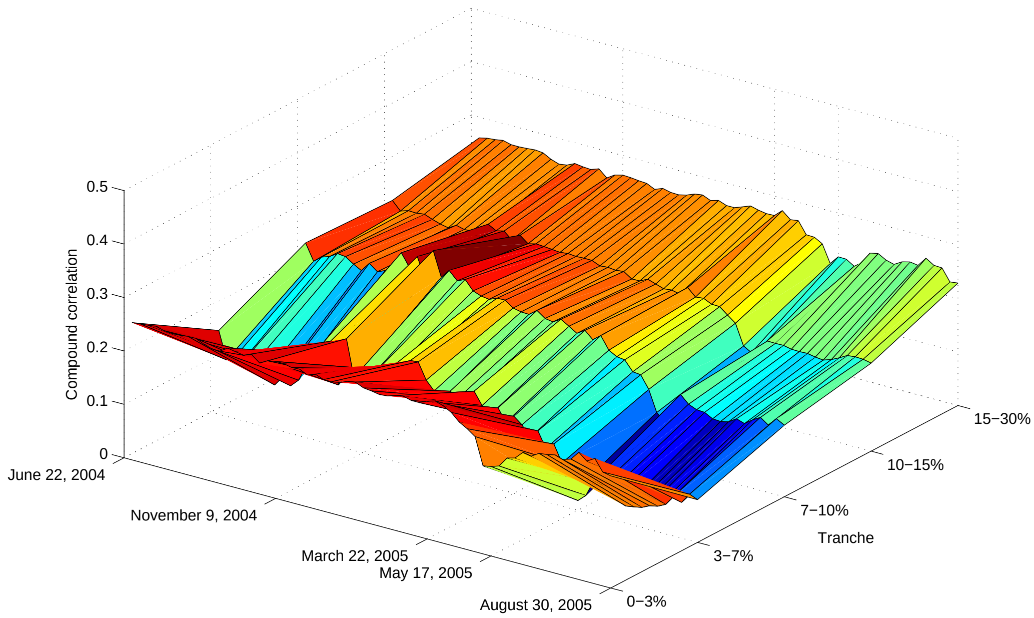

Figure 2: CDX.NA.IG 5Y: Compound correlation surface - Student-t Copula.

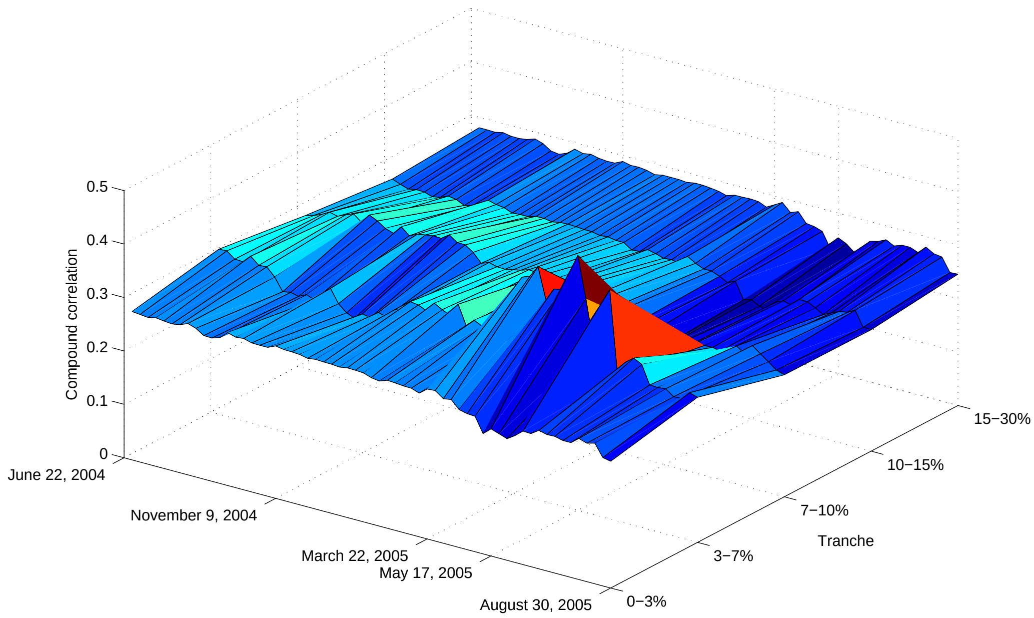

Figure 3: CDX.NA.IG 5Y: Compound correlation surface - STS Copula.

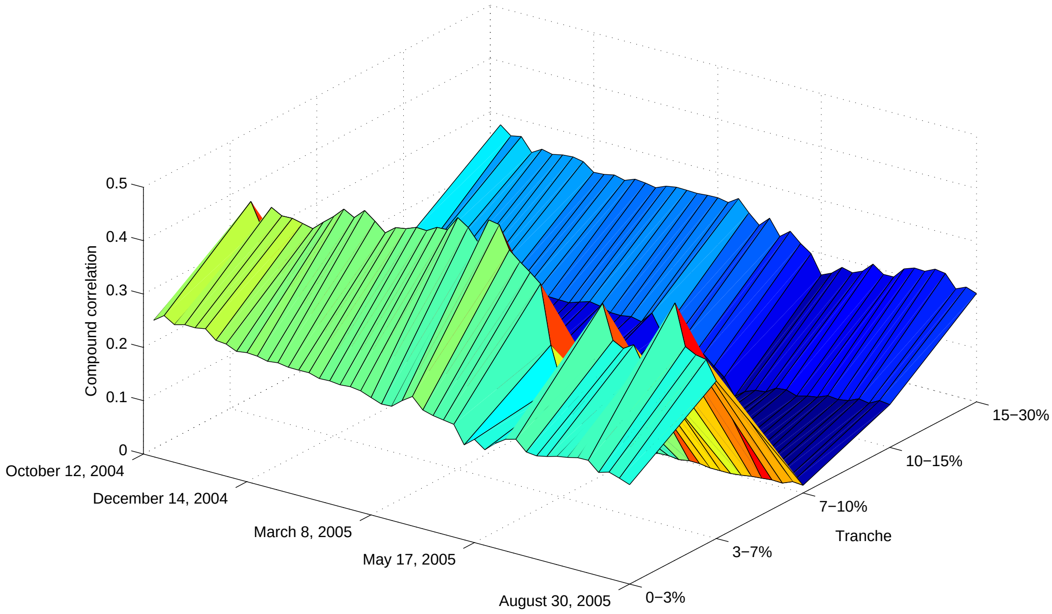

Figure 4: CDX.NA.IG 10Y: Compound correlation surface - Gaussian Copula.

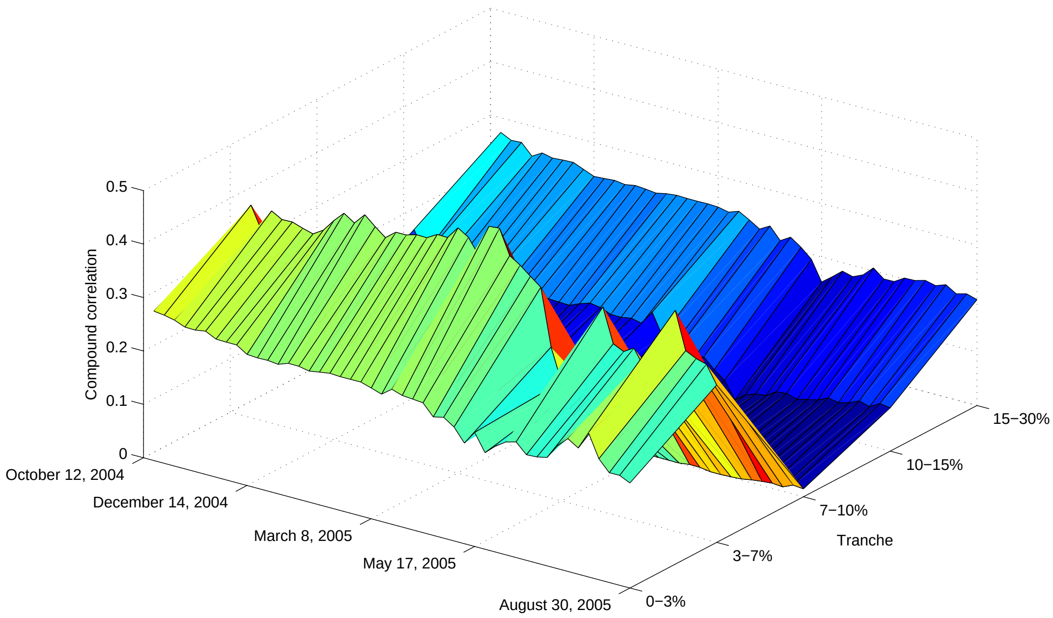

Figure 5: CDX.NA.IG 10Y: Compound correlation surface - Student-t Copula.

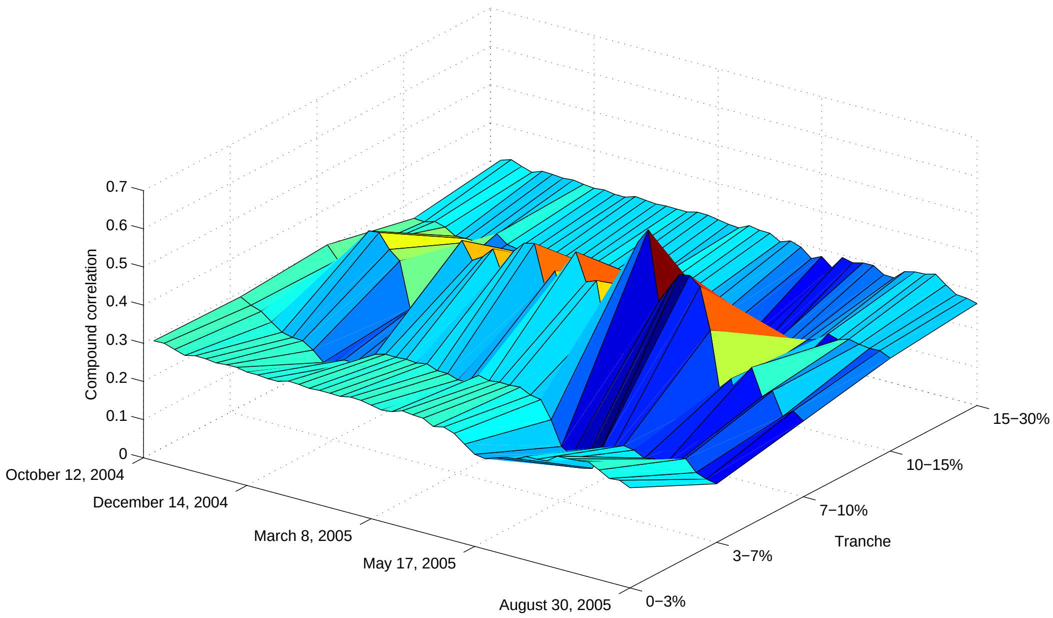

Figure 6: CDX.NA.IG 10Y: Compound correlation surface - STS Copula.

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