

# The Term Structure of Credit Spreads and Credit Default Swaps - an empirical investigation

STEFAN TRÜCK, MATTHIAS LAUB AND SVETLOZAR T. RACHEV

September 20, 2004

## Abstract

We investigate the term structure of credit spreads and credit default swaps for different rating categories. It is well-known quite that for issuers with lower credit quality higher spreads can be observed in the market and vice versa. However, empirical results on spreads for bonds with the same rating but different maturities are rather controversial. We provide empirical results on the term structure of credit spreads based on a large sample of Eurobonds and domestic bonds from EWU-countries. Further we investigate maturity effects on credit default swaps and compare the results to those of corporate bonds. We find that for both instruments a positive relationship between maturity and spreads could be observed for investment grade debt. For speculative grade debt the results are rather ambiguous. We also find that spreads for the same rating class and same maturity exhibit very high variation.

STEFAN TRÜCK<sup>1</sup>, MATTHIAS LAUB

Email: stefan@statistik.uni-karlsruhe.de, matthias.laub@gmx.de  
Institut für Statistik und Mathematische Wirtschaftstheorie  
Universität Karlsruhe  
Kollegium am Schloss, D-76128 Karlsruhe, Germany  
Fon: +49 (0) 721 608 8113, Fax: +49 (0) 721 608 3811

SVETLOZAR T. RACHEV<sup>2</sup>

Email: rachev@pstat.ucsb.edu  
Institut für Statistik und Mathematische Wirtschaftstheorie  
Universität Karlsruhe  
Kollegium am Schloss, D-76128 Karlsruhe, Germany  
and  
Department of Statistics and Applied Probability  
University of California, Santa Barbara, CA 93106, USA

---

<sup>1</sup>Corresponding author

<sup>2</sup>Rachev gratefully acknowledge research support by grants from Division of Mathematical, Life and Physical Sciences, College of Letters and Science, University of California, Santa Barbara, the Deutschen Forschungsgemeinschaft and the Deutscher Akademischer Austausch Dienst.

# 1 Introduction

Investing in bond markets always bears the potential risk of the loss of interest rate or principal payments, due to the fact that the issuer of the bond might not be able to meet his obligations. The probability of this scenario affects investment decisions for all market participants. Furthermore for individual financial contracts, like bank loans, potential lenders use it as one factor determining the contract specifications with respect to interest and maturity. Another aspect of credit risk is its meaning for the determination of the capital requirement for banks. The regulatory framework, set by the Bank for International Settlements (BIS), is revised currently. Initiated by the Basel Committee on Banking Supervision in 1999 it became known as “Basel-II”, whereas the most recent version titled “International Convergence of Capital Measurement and Capital Standards: a Revised Framework” (0) has been published in June 2004. One major novelty is the fact, that under certain restrictions banks are allowed to determine their capital requirement based on internal rating systems. All these aspects lead to the necessity of the development of reliable credit risk models, which support various decision-makers in the estimation and management of credit risk as well as in the pricing of financial instruments dealing with credit risk.

The main focus of the present paper is not the pricing of assets subject to credit risk, but are theoretical implications and empirical evidence regarding credit spreads. The credit spread is defined to be the additional amount of interest paid by a risky asset over the yield of a risk-free investment. In this context the term risky represents the credit risk, to which the asset is exposed through the probability of the issuer not being able to meet his obligations. This inability of meeting the obligation can be caused by insolvency, bankruptcy and further reasons leading to a delay or loss of promised payments and is referred to as the default of the obligor.

An interesting question is now, how the credit spread behaves depending on different factors. It is quite obvious, that for issuers exposed to a lower credit risk, i.e. the probability of default being less than for other issuers, the credit spread will be lower. But what kind of behavior for the credit spreads would be expected with varying maturity of the exposure, holding credit quality constant?

The answer to this question still seems to be controversial. Some empirical studies and observations result in a split behavior of credit spreads. They presume, that for high-grade bonds the credit spreads increase with maturity as for low-grade bonds they decrease, resulting in a downward-sloping risk structure. A brief summary of empirical studies can be found in section 3.1.

At a first glance the split behavior of credit spreads appears to be counter-intuitive, but an early approach to explain this effect has been undertaken by Johnson (0) and has become known as the “crisis-at-maturity-hypothesis”. It is argued, that speculative-grade companies with low credit quality, in his context identified as those having a high leverage-ratio, may face severe problems of refinancing as their short term debt matures. In consequence the risk of not being able to meet the obligations and thus the probability of default in the

short term is quite high. Once those companies have overcome their problems and survived a certain period of time without a default, they face a lower risk in the long run. For currently large and solid firms, the outlook in the short term is very stable with a low risk of default, whereas the forecast of credit quality over longer periods is less certain.

Is such a behavior reproducible by theoretical pricing models and can empirical evidence from market prices be found supporting these considerations? These questions are now examined, where the structure of the paper can be outlined as follows: First a few basic credit risk models and their implications for the term structure of credit spreads inherent in the individual model setting are presented in chapter 2. Although empirical work is very limited in this field, three former studies dealing with maturity effects of credit spreads are covered in chapter 3. Subsequently own empirical results based on a large sample of Eurobonds denominated in and domestic bonds from EWU-countries are presented.

The importance of questions concerning the measurement, hedging and trading of credit risk has provided the foundation for a strong development of the market for credit derivatives. After introducing this market and the main products, another empirical analysis is focussing on credit default swaps. This derivative instrument is designed to extract and transfer the pure default risk of a certain obligor and thus a strong relation to the default risk expressed in bond spreads is given. Therefore the presumed maturity effects are expected to appear in the market for credit default swaps as well. Chapter 4 examines this question and is followed by a conclusion of the presented work.

## 2 Credit Risk Models and their Implications for the Credit Spread

### 2.1 Structural Approach

#### 2.1.1 Merton-Model

Based on the equilibrium theory of option pricing, developed by Black and Scholes (0), Merton has builded up a pricing theory for corporate liabilities in general (0). The main idea behind his proceeding is the interpretation of corporate liabilities as options on the firm value.

Two basic ideas are building the background of Merton's model. First the default of a firm is determined by its value and is thus to a certain degree a foreseeable event. And second the event of default occurs, if the value of the firms assets  $V$  falls below the outstanding debt  $B$ .

Valuation of equity is carried out by applying the solution for the valuation of a European call option developed by Black and Scholes. Considering the connection of equity and liabilities by the accounting identity of the balance sheet, the value of the debt issue can be obtained and by furthermore using continuous compounding and incorporating the yield to maturity  $y(t, T)$ , a representation

for the credit spread is derived as

$$y(t, T) - r = -\frac{1}{T-t} * \ln\left(\frac{1}{d}\Phi(h_1) + \Phi(h_2)\right) \quad (2.1)$$

with

$$d = Be^{-r(T-t)}/V$$

and

$$h_1 = -\frac{\frac{1}{2}\sigma^2(T-t) - \ln(d)}{\sigma\sqrt{T-t}}$$

$$h_2 = -\frac{\frac{1}{2}\sigma^2(T-t) + \ln(d)}{\sigma\sqrt{T-t}}.$$

Now  $y(t, T) - r$  expresses the additional interest, which is paid by the risky issue compared to the riskless rate of interest  $r$ , the so called credit spread. The right side of equation (2.1) can be used to analyze the dependencies of the structure of credit spreads according to several factors.

Analyzing the effect of time to maturity  $T - t$  Merton highlights, that the change in the credit spread can be either sign, depending on  $d$ , which is referred to as the “quasi”-debt-to-firm-value ratio. This terminology is justified, because the debt is discounted at the risk-free rate of interest and therefore  $d$  is an upper bound for the true (market) debt-to-firm value ratio. He shows, that for  $d \geq 1$  the change of the premium will be negative, for  $d < 1$  it will be first positive, than negative, resulting in a so-called hump-shaped graph.

In two following notes, first Lee (0) and later Pitts and Selby (0) refined especially the graphical depiction of these dependencies, removing some inaccuracies incorporated in Merton’s paper. Despite this using the “quasi”-debt-to-firm-value ratio  $d$  as an indicator for credit quality, a split behavior in terms of the relationship between credit spreads and maturity is derived.

## 2.2 Reduced-Form Models

Reduced-form models are following an entirely different approach, modelling the default of a company as a rather unpredictable event. They do not rely on the value of the firm as an explanatory variable, but use external processes to represent occurring defaults. They use external ratings as one of the main sources of information, namely as the factor distinguishing the issuers with respect to credit quality.

### 2.2.1 Fons (1994)

In 1994 Jerome S. Fons published an article (0) addressing the term structure of credit spreads, which can be seen as one of the first reduced-form models. The only source of information included in the model are historical default probabilities, rating information and an estimate for the recovery rate. The recovery rate  $\mu$  expresses the percentage of the exposure, which the investors can expect to receive in the case of default.

The cumulative probability  $cpd$  of default for a specific rating category  $R$  and a time horizon of  $t$  years reflects the probability, that an issue defaults up to year  $t$  after holding the rating  $R$ . The marginal default probability  $mpd$  in year  $t$  after holding credit rating  $R$  is defined to be the difference in cumulative probabilities of year  $t$  and  $t - 1$ . The forward probability of default  $fpd$  is now defined as the probability of defaulting in year  $t$  after holding the rating, given that default has not occurred up to time  $t - 1$  and can therefore be identified as a conditional probability of default. By verifying, that all those probabilities are equal for  $t = 1$  they can be mapped to each other in the following way:

$$\begin{aligned} mpd_R(t) &= cpd_R(t) - cpd_R(t - 1) \\ fpd_R(t) &= \frac{mpd_R(t)}{1 - cpd_R(t-1)} \end{aligned}$$

To simplify the representation in subsequent formulas, the cumulative survival rate should be introduced, calculated simply as  $S_R(t) = 1 - cpd_R(t)$ .

Now this data is used to develop a model for corporate bond pricing and for explaining observable credit spreads. The original version of Fons' model is transferred to the world of discount bonds, i.e. bonds not paying a periodical coupon. Furthermore using continuous compounding, a simple presentation of the credit spread can be obtained.

The price for a security exposed to default risk, including the credit spread  $s$ , can be expressed as

$$B(0, T) = Be^{-(r+s)T}. \quad (2.2)$$

On the other hand the assumption of risk-neutral investors leads to the price being the expected value of the payoffs received from the asset. As the only payoff of a zero-coupon bond takes place at maturity, for every point of time  $t < T$  only the case of default with the recovered fraction of the face value  $B$  has to be incorporated. Together with the notation for the different default and survival probabilities as introduced above, the credit spread  $s$  can now be obtained as

$$s = -\frac{1}{T} \ln \left( \sum_{t=1}^T S_R(t-1) fpd_R(t) \mu B e^{-r(t-T)} + S_R(T) \right) \quad (2.3)$$

In the next step the term-structure, i.e. the behavior of the spread with respect to different maturities, can be analyzed. Fons applies a constant recovery rate of  $\mu = 48.38\%$ , the long term average for senior unsecured issues as reported by Moody's and the riskless rates are obtained by fitting a regression model to the U.S. Treasury schedule of September 30, 1993.

As a result of the spread calculation according to his model, Fons discovers an almost strictly upward slope for the credit spreads for bonds in investment-grade classes. The so-called hump-shaped behavior, which already was proposed by the model of Merton, can be observed for the rating BB. Credit spreads are increasing up to a maturity of 5 years and decrease afterwards. For the rating class B a strictly downward slope has been calculated.

Now that the model provided us with results supporting the split behavior of the term structure of credit spreads, the question is, how this evidence can be found in spreads observable at the market and how good market spreads are estimated by the model. For the reference date of September 30th, 1993 the examination of yields of roughly 2850 U.S. corporate bonds provided the following results:

- For rating class AAA no systematic change in credit spreads with increasing maturity can be observed.
- For rating classes AA and A a significant positive slope coefficient for the regression can be found, which means that in these rating classes the credit spread rises as maturity increases. Even though the effect is a little less strong, for rating class BBB this positive relationship between spread and maturity can be identified as well.
- Although regression provides a negative slope coefficient for rating class BB, the required significance is not assured.
- Finally for rating class B the plot indicates a negative spread–maturity relationship, supported by a significant negative slope coefficient of the corresponding regression.

Comparing the theoretical results with the market spread it has to be mentioned, that spreads received from the pricing equation are basically and essentially lower than market spreads. This is in particular true for investment–grade rating classes, whereas calculated spreads in the speculative–grade classes more closely fit observable market spreads.

Nevertheless it is remarkable that Fons is able to explain the structure of the credit spreads for the reference date with this basic model, using only a recovery rate estimate, rating information and historical default probabilities.

### **2.2.2 Jarrow, Lando, Turnbull (1997)**

The model developed by Jarro, Lando and Turnbull (0) in 1997 belongs to the class of intensity models and incorporates the probabilities of rating transitions into the valuation process. Markets are assumed to be complete and free of arbitrage opportunities and the process of rating transitions and interest rates are independent under the martingale measure  $\mathbb{Q}$ . The time–homogeneity of the transition matrix is imposed and default times are exponentially distributed with parameter  $\lambda$ . This assumption leads to the default being the first occurrence of a homogeneous Poisson–process with time–independent intensity  $\lambda$ .

Furthermore the default intensity  $\lambda$  becomes explicitly dependent upon the credit rating of the issue. For the representation of changes in credit quality through probabilities of rating changes a time–homogenous Markov chain is chosen. In the discrete time case, the one period transition matrix  $\mathcal{Q}$  contains the transition probabilities between the possible states. Market prices are used to extract time– and state–dependent risk premia, which in the next step

transform the matrix of transition probabilities to time-dependent risk-neutral matrices  $\tilde{Q}_{t,t+1}$ . They are used to calculate default and survival probabilities for all maturities.

An expression for the credit spread in terms of the spot rates, depending on the recovery rate  $\mu$  and the martingale rating transition probabilities is obtained as

$$r^R(t) - r'(t) = -\log(1 - (1 - \mu)\tilde{q}_{iK}(t, t+1)) \quad (2.4)$$

Following a comparable argumentation, the corresponding results can be elaborated for the continuous time case. Refer to (0) for a detailed description of the proceeding.

In the empirical part of the paper survival probabilities and spreads under risk neutrality are presented and analyzed. Risk neutrality in this context means, that the empirical transition probabilities are used for pricing and no adjustment by the use of any risk premium is undertaken. Furthermore the recovery rate is set to zero, thus allowing another interpretation of the credit spread as a hazard rate. The hazard rate  $\tilde{\lambda}_{iK}(t)$  is the default rate at time  $t$  of an issuer rated  $i$  at time 0, which has not defaulted up to time  $t$ . From the results presented in the article, a few important effects can be observed. For issues rated AAA, AA or A, a strictly upward sloping credit spread curve is derived. Within a time horizon of 30 years, BBB rated issues are the first, for which a hump-shaped curve is obtained. While BB rated issues are showing a comparable behavior, the curve for class B decreases starting with the second year. Finally for rating class CCC a strictly negative spread-maturity relationship can be observed. The extraordinary high spreads in the subinvestment categories and the fact, that the spread curve for class CCC even falls below the curve for class B imply inconsistencies. Nevertheless this model is able to generate the split behavior of credit spreads with respect to maturity as well.

Starting from the basic concepts introduced in the reduced-form models, the development of this model class is still going on<sup>3</sup>.

### 3 Evidence from the Bond Markets

#### 3.1 Former Empirical Studies

Having laid the theoretical foundations for the analysis of maturity effects in credit spreads, a few empirical studies should be presented.

The empirical study of Sarig and Warga (0) investigates the term structure of credit spreads for a set of pure discount-bonds and discusses the findings in relation to the theoretical behavior suggested in the firm value approach by Merton (0). The sample covers the time period from February 1985 to September 1987 and consists of 137 corporate zero-bonds, issued by 42 different companies. The average of yield spreads for the bonds in a given month is calculated

---

<sup>3</sup>For a detailed survey of standard reduced-form and structural credit risk models and their extensions refer to (0)

and afterwards averaged across time. The authors retrieve a downward-sloping term structure of credit spreads for bonds rated B or C, a hump-shaped behavior for rating class BB and an upward-slope for the investment grade classes. These results are now compared with the theoretical behavior of credit spreads according to the structural model by Merton and are interpreted as a confirmation. To apply the argumentation proposed in the theoretical model, a negative correlation between rating and “quasi”-debt-to-firm-value ratio  $d$  has to be assumed.

Another study, providing a different thesis regarding maturity effects in credit spreads, was conducted by Helwege and Turner in 1998. The argumentation of Helwege and Turner (0) is based on an assumed selection bias, evolving from the fact, that issuers with the same credit rating are treated equally. They argue that, especially for the subinvestment-grade classes, credit quality within one rating class varies significantly. Assuming, that the more creditworthy issuers within one rating class tend to issue debt with longer maturities, the credit yield curve will be biased downward for this rating category.

The sample includes 64 straight public U.S. subinvestment-grade bond offerings and 163 bonds from 1977 to 1994, ranked equally in the priority structure. Only so-called “matched cases”, that is groups of bonds from the same obligor, issued on the same day are examined. In the first step of the analysis, for every single issuer the credit spread of the bonds issued at the same day, is examined. Regardless of the absolute difference of the spread or of the maturity it is observed, if the spread rises or decreases with maturity. While 77 % of the matches are strictly upward sloping, the positive spread-maturity relation for the subinvestment-grade classes is supported by the nonparametric Wilcoxon signed rank test as well as the  $t$ -test.

One further study dealing with credit spreads and credit quality as their only explaining factor is a publication of Jeffrey Bohn (0). Covering the time period between June 1992 and January 1999 data from more than 24000 bonds denominated in U.S.-\$ and more than 1700 issuers results in a sample size of almost 600000 observations.

The key difference in the proceeding of Bohn is the classification of issues according to credit quality. While usually the credit rating is used to group issues with comparable credit quality, Bohn additionally uses the Expected Default Frequency. This measure for the probability of default over a specific time horizon is provided by KMV corporation and can be seen as one practical implementation of a structural firm value model in the spirit of Merton. The one-year EDF as well as the geometric mean of all one to five year EDF's is used to classify the issues according to credit quality. One major advantage of EDF's instead of credit ratings is the fact, that they can be calculated for each date of the sample period. Therefore they reflect current credit quality more precisely than credit ratings, which are adjusted not that frequent.

Bohn chooses a special way to deal with the question of maturity of the assets. He does not group issues according to equal maturity, but uses Macaulay duration as a measure for classification. By calculating the cash-weighted average time-to-maturity the coupon effect is incorporated and thus duration can



Rating	Time Horizon [years]									
	1	2	3	4	5	6	7	8	9	10
AAA	0.00	0.00	0.04	0.03	0.05	0.09	0.10	0.18	0.07	0.07
AA	0.01	0.03	0.07	0.09	0.13	0.15	0.20	0.17	0.15	0.18
A	0.05	0.10	0.15	0.20	0.25	0.26	0.28	0.26	0.34	0.33
BBB	0.37	0.69	0.75	1.06	1.03	1.03	0.87	0.81	0.74	0.96
BB	1.45	2.97	3.76	3.71	3.45	3.73	3.15	2.86	3.06	2.45
B	6.59	9.04	8.74	7.75	6.36	5.81	5.59	5.52	3.91	4.38
CCC	34.14	15.08	11.57	10.33	12.85	6.68	2.8	1.83	9.30	8.59

Table 1: Average forward default rates [%] 1981 – 2003

be used to form homogeneous groups of bonds. Using data from June 1992 to January 1999, monthly medians of credit spreads for each maturity bucket and credit quality class are calculated and averaged across time. The spread curves generated from this analysis show a comparable behavior to those presented by Sarig and Warga (0): A positive slope for the investment-grade classes and a hump-shaped or downward-sloping term structure for issues of currently low credit quality. Even though the sample includes a large number of observations, a few anomalies like the crossing of spread curves representing different credit quality, can be observed. This effect becomes exceedingly evident in the case where a snapshot at a specific point of time is examined.

Reflecting the results of the former studies, in particular the controversial observations and the overall heterogeneity of data, the question might arise, if default risk is really the main factor determining the credit spread of an issue. Liquidity, tax effects and market risk factors may have significant influence on spreads. Those questions are examined in an empirical study of Delianedis and Geske (0) as well as by Elton, Gruber, Agrawal and Mann (0). Both conclude, that especially in the investment-grade rating classes default risk is outweighed by those factors, whereas with a decline in credit quality the contribution of default risk to the credit spread rises.

## 3.2 Empirical Results for the Term Structure of Bond Spreads

### 3.2.1 A First Glance at Conditional Probabilities of Default

After discussing the theoretical background, models from different classes dealing with credit risk and the presentation of former empirical studies, several proprietary results should be outlined. In the first step historical default rates as reported by rating agencies are examined. The source of data for the analysis in this step is a publication from Standard & Poor's (0), where cumulative average default rates are provided, covering the time period 1981 to 2003. Based on these rates, the average forward default rates are calculated and presented in table 1.

A graphical representation of the behavior of forward-pd's with respect to the time horizon is given in figures 1 and 2.

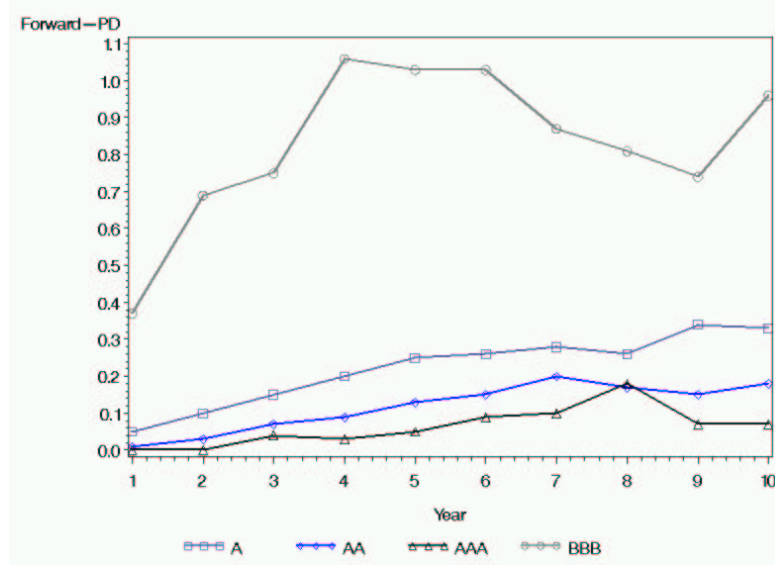


Figure 1: Forward-pd's for investment-grade rating classes

For the three rating categories AAA, AA and A an obvious increase of the forward-pd can be determined, whereas for rating class BBB, BB and B a hump-shaped progression can be observed. For issues rated CCC the shape of the term structure is decreasing from the first year. The split behavior of forward-pd's, which is suggested by the considerations of the "crisis-at-maturity-hypothesis" can be identified. This can be seen as a first indication for credit spreads behaving in a similar manner, which should be investigated in the following chapter.

### 3.2.2 Description of the Bond Sample

The Reuters Eurobond pages and the national pages from EWU countries were the main source of information, where only issues denominated in have been extracted. The unique identifier for each asset was the International Securities Identification Number (ISIN). One internal database of a major German bank, including a comprehensive sample of about 800 -denominated bond issues, a publication of JPMorgan (0) and the search function of onvista<sup>4</sup> have been used to obtain additional assets by extracting their ISIN. In the next step static information like coupon, coupon frequency, maturity, optional features, rating and several further characteristic attributes have been extracted from Reuters as well as from Bloomberg. This data has then been cross-checked to eliminate erroneous information. After eliminating all floating rate notes, a set of

<sup>4</sup><http://anleihen.onvista.de/suche-vergleich/unternehmen.html>

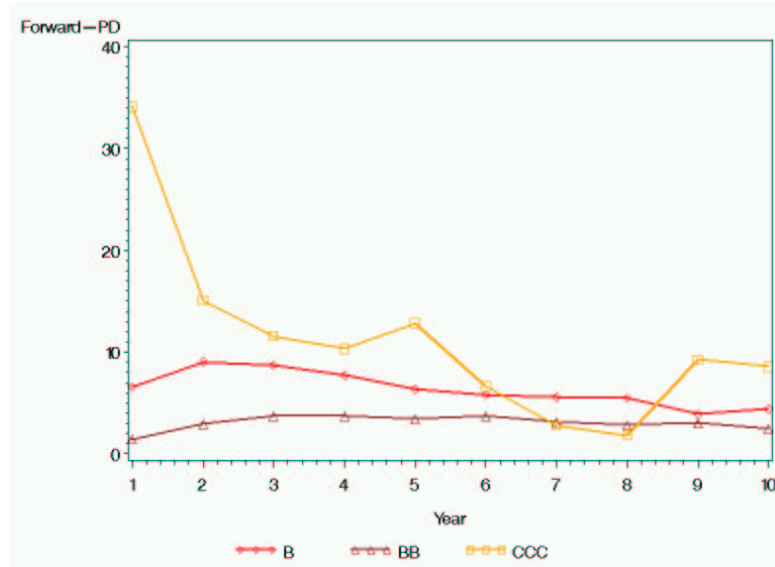


Figure 2: Forward-pd's for subinvestment-grade rating classes

2400 bonds remained, for which historical price and yield information has been obtained from Bloomberg. Comparable to the former studies and to assure sufficient quantity of data especially for longer maturities, issues with a time to maturity between 0.5 and 10.5 years were included in the evaluation. Furthermore this assures an equal length of each time interval while grouping the assets to integer maturity ranges. Subsequently the sample has been restricted to bonds paying annual, constant coupon rates. Likewise all issues incorporating any kind of optional feature like callability, putability or convertibility have been excluded from further research. Depending on the availability of a price and yield information for the desired reference date, approximately 2000 bonds are available for examination.

### 3.2.3 Risk-Free Term Structure and Spread Calculation

The yields of German and French government securities, provided by Bloomberg, are used to generate a term structure of interest rates, which can be referred to as being risk-free. Linear interpolation is carried out to calculate risk-free quotes for 3 month, 6 month, 1, 2, ..., 10, 15, 20 and 30 years on a daily basis. Once this risk-free term structure is provided, the spread calculation for every single asset and reference day can be accomplished. Equipped with the historical yield information for every asset on a daily basis, the observable market credit spread is obtained as the difference between the asset's yield and the corresponding risk-free rate. Again a linear interpolation is applied, as maturities of the assets usually do not equal the integer time horizons for which risk-free quotes are available.

	n	$\rho$	b	t-stat.	p-value	$R^2$
AAA	707	0.101	0.003	2.7	< 0.0001	0.010
AA	426	0.188	0.009	3.93	< 0.0001	0.035
A	595	0.253	0.018	6.37	< 0.0001	0.064
BBB	334	0.231	0.039	4.34	< 0.0001	0.054
BB	46	0.316	0.139	2.21	< 0.0001	0.099
B	40	0.328	0.192	2.14	0.039	0.108

Table 2: Correlation and regression results for bondspreads as of February 11th, 2004

### 3.2.4 Results for February 11th, 2004

February 11th, 2004 is chosen as the first reference date for the examination of market spreads. Figures 3 to 8 show the spreads based on the single issues and classified according to the different rating classes. For each rating class the Pearson correlation coefficient  $\rho$  and the parameters of a linear regression model are provided in table 2.

Although the slope coefficients are positive and significantly different from zero for all rating categories, the overall fit of the regression is very low, as indicated <sup>5</sup>by the value of  $R^2$ . For AAA-rated issues two properties are immediately observable. First, a maturity effect can almost not be identified. This contradicts theoretical predictions, but corresponds with the empirical results of Fons. Considering the findings, that especially in the high rating classes the default risk might account only for a smaller fraction of the credit spread, the observed results can be justified. The second remarkable observation is the occurrence of negative credit spreads. This heavily depends on the choice of the risk-free reference. Generally LIBOR is known to be comparable to a AA-rated security. Therefore a negative spread between LIBOR and AAA-rated government or corporate bonds can be expected. Nevertheless in the present case, where AAA-rated government securities are used to create the risk-free structure, this should not occur. The question is, if the German and French securities are appropriate benchmarks for the examined bond sample and to which extend the linear interpolation causes inconsistent results. Although the occurrence of the negative spreads should not be neglected, the overall number of issues showing this behavior is small compared to the sample size.

AA-rated issues exhibit a higher credit spread and the number of negative spreads further decreases. Supported by a higher slope coefficient of the regression, a positive relationship between maturity and spread can be observed, although the overall variation is extraordinary high and interpretation of the maturity effect and further implications seem to be questionable. The same holds for A- and BBB- rated issues, although the positive spread-maturity relationship becomes more apparent. Caused by a lack in data quantity, con-

---

<sup>5</sup>for a discussion of problems of using the  $R^2$ -statistic as a measure of fit for a regression model, refer to appendix A

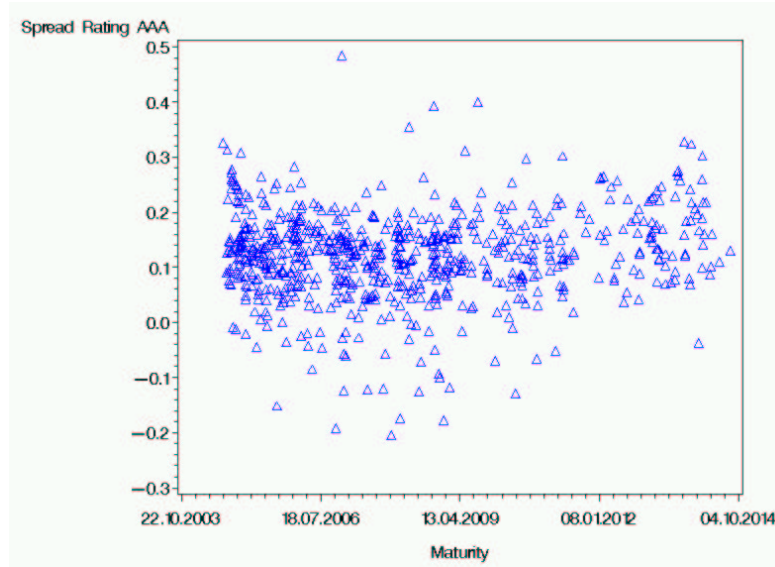


Figure 3: Bondspreads as of February 11th, 2004 for AAA-rated issues

clusions become even more vague for BB- and B-rated issues. Although the correlation indicates a positive relationship as well, it is obvious, that a small number of data points has a major influence on all results and the robustness of all results is very low. Nevertheless the results do not provide any indication for the existence of a downward-slope in the term structure of credit spreads.

In the next step, the raw data is aggregated to highlight some of the key results. The assets are grouped to integer maturity buckets, where all assets with a time to maturity between 0.5 and 1.5 years are subsumed to maturity 1, all assets between 1.5 and 2.5 years to maturity 2 and so on. The average spread for the different rating classes is represented in figure 9 for investment-grade and in figure 10 for subinvestment-grade rating classes.

The general tendency of a positive spread-maturity-relationship for the investment-grade rating classes is supported by this representation, whereas the noisiness of the results becomes in particular obvious for BB- and B-rated issues. In table 3 the correlation and regression results for the average bondspreads are presented, confirming these considerations. For the investment-grade rating categories a significantly positive slope coefficient is obtained, together with reasonable high values of the  $R^2$ -statistic between 47% and 87%.

In table 6, mean, standard deviation and coefficient of variation is presented for every rating class and maturity bucket. By measuring the variability of the spreads for each rating class and maturity bucket via the coefficient of variation, additional observations can be interpreted. In general the overall heterogeneity of the data sample is supported, whereas the spreads for AAA-rated issues exhibit a high relative variation in particular. Again the quantity of data for

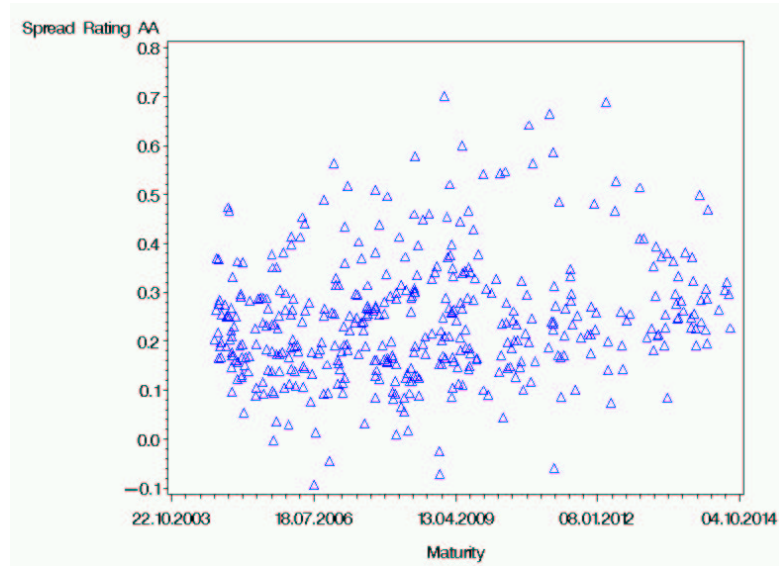


Figure 4: Bondsreads as of February 11th, 2004 for AA-rated issues

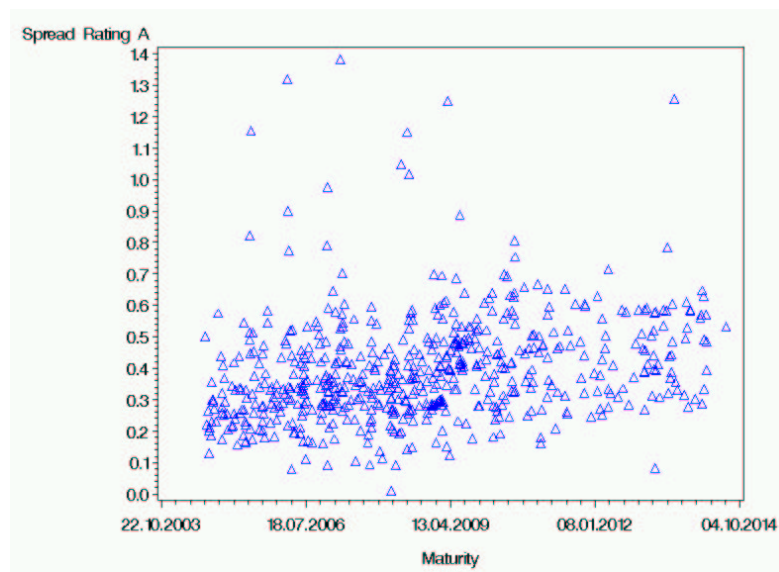


Figure 5: Bondsreads as of February 11th, 2004 for A-rated issues

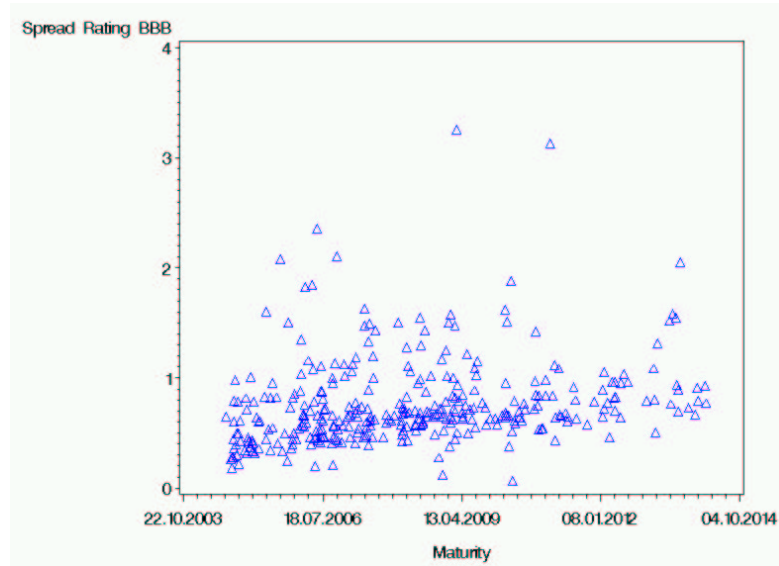


Figure 6: Bondsreads as of February 11th, 2004 for BBB-rated issues

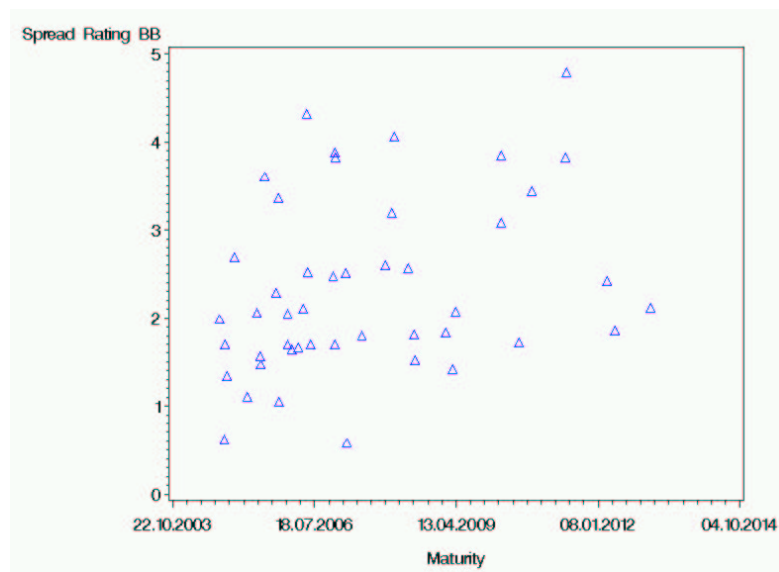


Figure 7: Bondsreads as of February 11th, 2004 for BB-rated issues

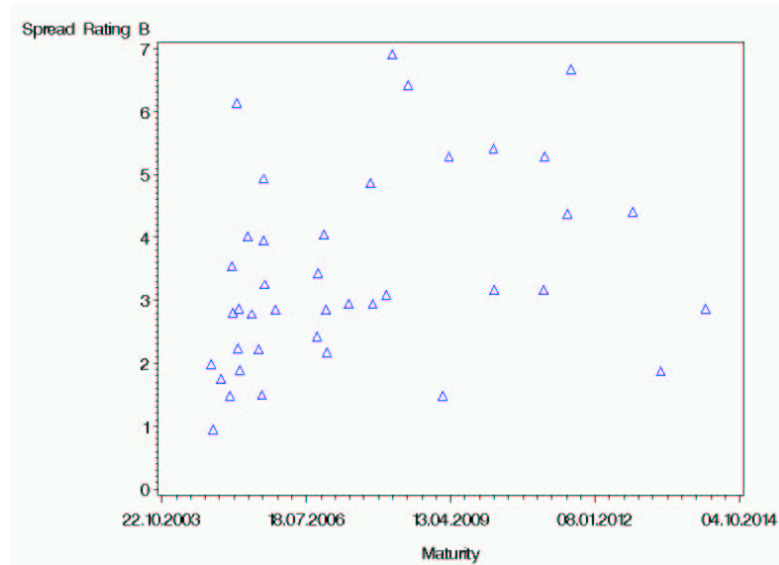


Figure 8: Bondsreads as of February 11th, 2004 for B-rated issues

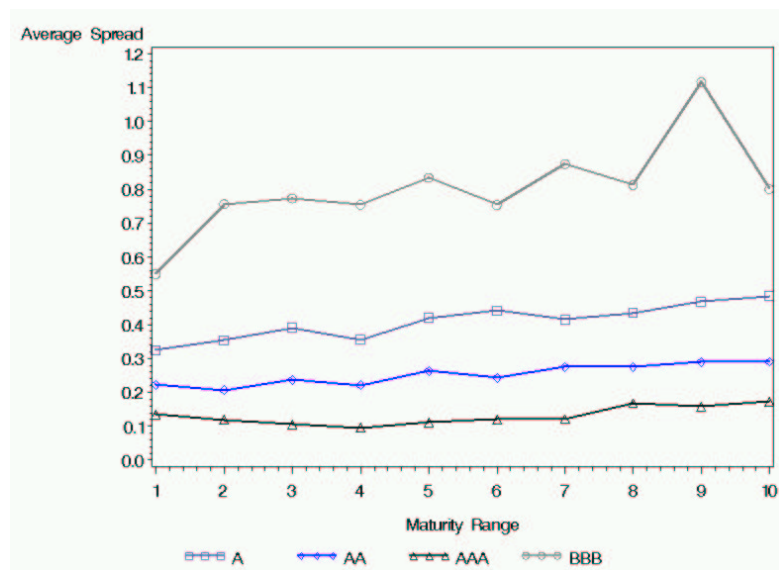


Figure 9: Average bondsreads as of February 11th, 2004 for investment-grade rated issues



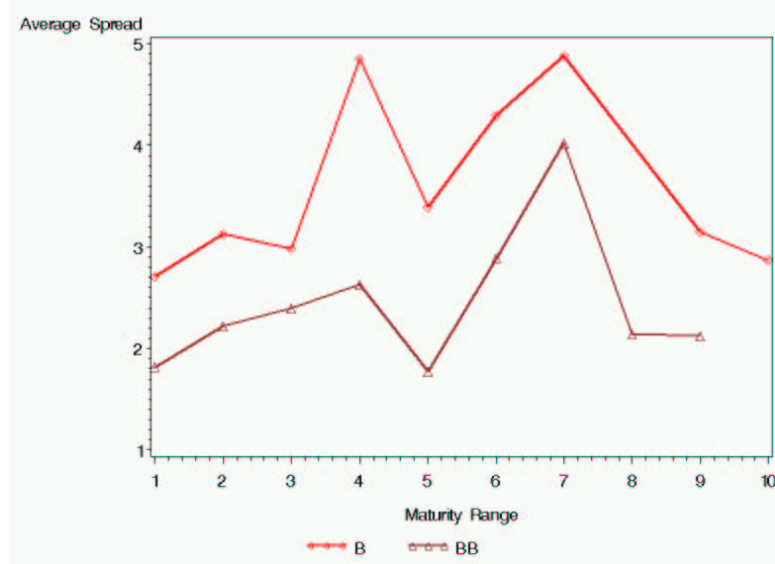


Figure 10: Average bondsreads as of February 11th, 2004 for subinvestment-grade rated issues

	n	$\rho$	b	t-stat.	p-value	R <sup>2</sup>
AAA	10	0.692	0.006	2.71	0.0001	0.479
AA	10	0.924	0.009	6.85	0.0001	0.854
A	10	0.937	0.016	7.6	<0.0001	0.878
BBB	10	0.689	0.032	2.69	<0.0001	0.475
BB	9	0.297	0.075	0.82	0.438	0.088
B	9	0.133	0.037	0.35	0.733	0.018

Table 3: Correlation and Regression results for maturity buckets of bondsreads as of February 11th, 2004

the subinvestment-grade rating classes prevents a detailed inspection.

A further issue of the examination of bondspreads deals with the comparison of observed market spreads and calculated spreads, based on the forward-pd's. For this purpose the expected spreads are derived according to the formula presented by Fons in section 2.2.1. The assumption of a flat term structure is relaxed and the given term structure is used instead. With  $r_t$  being the non-stochastic risk-free rate for an investment with time to maturity  $t$ , the formula for the spread calculation becomes

$$s = -\frac{1}{T} \ln \left( \sum_{t=1}^T S_R(t-1) f_{pd_R}(t) \mu B e^{-r_t t + r_T T} + S_R(T) \right) \quad (3.1)$$

The question of the appropriate recovery rate is addressed as follows. With the available information for the bonds regarding priority in capital structure, it was possible to identify 1175 senior secured, 909 senior unsecured and 71 junior issues. To assure a sufficient quantity of data in all rating classes and for all maturities, the sample is not split, but a weighted average recovery rate is calculated. Based on the historical, value-weighted recovery rates as reported by Moody's (0), a value of  $\mu=0.486$  is obtained. The results of the spread calculation are shown in table 4 and a graphical representation is given in figure 11 and 12.

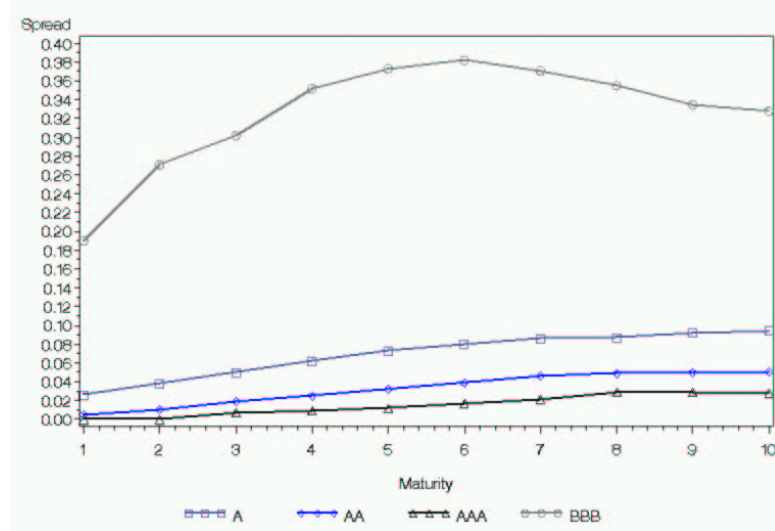


Figure 11: Calculated bondspreads for February 11th, 2004

All the before mentioned considerations and theoretical predictions regarding maturity effects, are illustrated. The positive slope for the investment-grade rating categories, a hump-shaped progression starting with class BBB and the strictly negative slope for the lowest rating class is obvious. By comparing the calculated spreads with the market spreads it is observable, that for

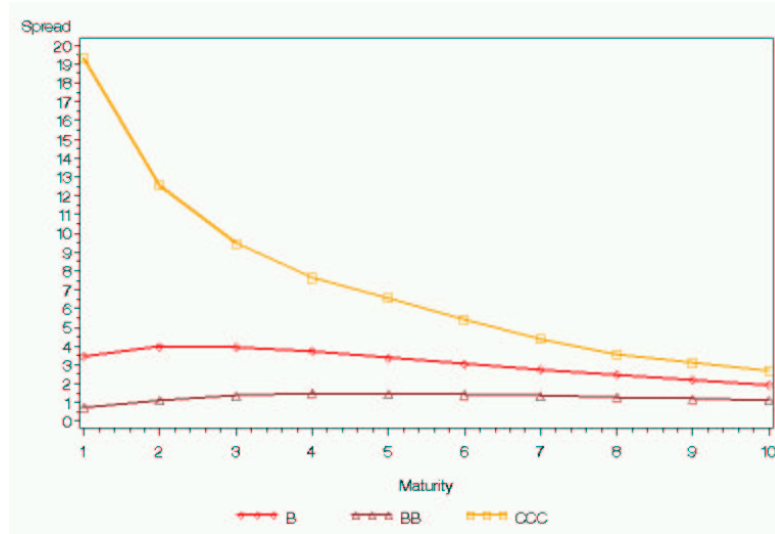


Figure 12: Calculated bondspreads for February 11th, 2004

the investment-grade rating categories the calculated spreads are far below the market spreads, whereas they match more closely in the subinvestment-grade rating categories. These findings are consistent with the results of Fons. One possible reason for this behavior is the fact, that the probability of default may only account for a smaller fraction of the credit spread in the investment-grade rating categories, as it has been discussed in section 3.1. In a subsequent step, the calculated spreads are used as the explanatory variable for the observed market spreads in a regression model. The results are given in table 5. Statistically significant results are obtained for rating classes AAA, AA and A. The explanatory power of the regression as indicated by  $R^2$  lies between 50% and 84%. Both a positive intercept and a positive slope coefficient reflect the fact, that the market spreads are higher than the calculated spreads.

### 3.2.5 Results for August 11th, 2003

The same analysis is conducted for a second reference date, August 11th, 2003. All related graphics and analytical results are presented in appendix B. Regarding the maturity effect, the results for the second date can be summarized as follows:

- A positive spread-maturity relationship is observed for rating classes AA, A, BBB and B. For AAA and BB the required statistical significance is lacking.
- No evidence for a negative spread-maturity relationship is found, although limitations in data quantity call results for the subinvestment-grade rating classes into question.

Rating	Time Horizon [years]									
	1	2	3	4	5	6	7	8	9	10
AAA	0	0	0.007	0.009	0.012	0.017	0.021	0.029	0.029	0.028
AA	0.005	0.01	0.019	0.025	0.032	0.039	0.046	0.049	0.05	0.051
A	0.026	0.038	0.05	0.062	0.073	0.08	0.086	0.087	0.092	0.094
BBB	0.19	0.271	0.302	0.352	0.373	0.382	0.371	0.355	0.335	0.328
BB	0.748	1.128	1.362	1.513	1.43	1.423	1.355	1.27	1.2	1.103
B	3.446	3.971	3.955	3.717	3.369	3.042	2.748	2.486	2.19	1.947
CCC	19.295	12.543	9.468	7.627	6.548	5.402	4.381	3.563	3.11	2.686

Table 4: Calculated spreads [%], based on forward-pd's, for February 11th, 2004

	n	$\rho$	a	t-stat	p-value	b	t-stat	p-value	R <sup>2</sup>
AAA	10	0.713	0.105	9.59	<.0001	1.703	2.88	0.021	0.509
AA	10	0.914	0.2	21.58	<.0001	1.62	6.36	0.0002	0.835
A	10	0.922	0.271	12.44	<.0001	2.015	6.71	0.0002	0.849
BBB	10	0.561	0.367	1.59	0.151	1.339	1.92	0.091	0.315
BB	9	0.409	0.883	0.66	0.53	1.038	1.19	0.275	0.167
B	9	0.048	3.408	2.4	0.048	0.056	0.13	0.903	0.002

Table 5: Regression of bond market spreads on calculated spreads for February 11th, 2004

	Time Horizon [years]									
	1	2	3	4	5	6	7	8	9	10
	AAA									
$\mu$	0.14	0.12	0.11	0.1	0.11	0.12	0.12	0.17	0.16	0.17
$\sigma$	0.07	0.08	0.08	0.09	0.08	0.09	0.07	0.07	0.06	0.08
$\sigma/\mu$	0.52	0.64	0.78	0.92	0.74	0.72	0.56	0.43	0.38	0.47
n	137	107	101	86	87	53	44	22	41	29
	AA									
$\mu$	0.22	0.21	0.24	0.22	0.26	0.24	0.27	0.28	0.29	0.29
$\sigma$	0.09	0.12	0.11	0.13	0.14	0.13	0.16	0.15	0.1	0.08
$\sigma/\mu$	0.39	0.56	0.48	0.57	0.53	0.52	0.6	0.56	0.33	0.27
n	60	57	53	65	59	35	31	19	24	23
	A									
$\mu$	0.32	0.35	0.39	0.36	0.42	0.44	0.42	0.44	0.47	0.48
$\sigma$	0.16	0.17	0.18	0.18	0.16	0.14	0.13	0.14	0.18	0.13
$\sigma/\mu$	0.5	0.49	0.46	0.5	0.38	0.33	0.32	0.31	0.39	0.26
n	63	76	85	89	92	66	40	27	38	19
	BBB									
$\mu$	0.55	0.76	0.77	0.75	0.83	0.75	0.88	0.81	1.12	0.8
$\sigma$	0.27	0.45	0.38	0.29	0.46	0.38	0.52	0.17	0.45	0.1
$\sigma/\mu$	0.49	0.6	0.49	0.38	0.55	0.5	0.59	0.21	0.4	0.13
n	46	54	54	43	50	26	25	17	13	6
	BB									
$\mu$	1.82	2.22	2.4	2.63	1.78	2.89	4.02	2.14	2.12	–
$\sigma$	0.84	0.92	1.18	0.93	0.33	1.08	0.7	0.4	0	–
$\sigma/\mu$	0.46	0.41	0.49	0.35	0.19	0.37	0.17	0.19	0	–
n	10	11	7	6	3	3	3	2	1	–
	B									
$\mu$	2.71	3.12	2.98	4.85	3.39	4.29	4.88	0	3.15	2.87
$\sigma$	1.39	1.23	0.68	1.84	2.7	1.6	1.48	0	1.8	0
$\sigma/\mu$	0.51	0.39	0.23	0.38	0.8	0.37	0.3	0	0.57	0
n	12	6	6	5	2	2	4	0	2	1

Table 6: Statistical indicators for bondspreads as of February 11th, 2004

- The overall fit of the regression models is extremely low for all rating classes as indicated by the  $R^2$ -statistic<sup>6</sup>.

---

<sup>6</sup>consider appendix A

Overall the behavior of spreads is comparable, whereas for August 11th, 2003 they are generally higher than for February 11th, 2004. The relative variability as measured by the coefficient of variation does not exhibit a regular behavior, apart from the fact that it is generally lower for rating class AAA compared to the other reference date.

## 4 Credit Derivatives

### 4.1 The Market for Credit Derivative Products

Credit derivative products are designed to isolate specific aspects of credit risk from one or more underlying assets. They are used to transfer these risks between the contract partners, which allows to actively trade and hedge credit risk. The major credit derivative products are Total Return Swaps (TRS), Credit Spread Options (CSO), Collateralized Debt Obligations (CDO), Credit Linked Notes (CLN) and Credit Default Swaps (CDS). A detailed explanation of those contracts can be found for instance under <http://www.creditderivatives.cc>, whereas it has to be mentioned that the market for credit derivatives is extraordinary innovative and the development of new products permanently continues.

#### 4.1.1 Credit Default Swap (CDS)

A Credit Default Swap is a financial contract, where the protection buyer pays a periodical fee on the notional amount of the reference underlying. In return the protection seller is committed to effect a default payment, in case a credit event with respect to the underlying reference entity occurs. In a single name CDS this underlying will usually be one specific bond, whereas in a basket CDS the underlying consists of a portfolio of credit risky assets. The protection buyer still faces the risk of a change in credit quality and market value of the underlying, whereas the risk of default is isolated in this contract and transferred to the protection seller. Therefore it is intuitive to compare the CDS premium, expressed in basis points per year, with the credit spread on the market of the underlying. Figure 13 shows the setting of a theoretical risk-free trade, where the investor has perfectly hedged the default risk. For simplicity it is assumed, that the asset is issued at par and the credit spread is obtained as the difference of the coupon and the risk-free rate  $r$ . From no-arbitrage considerations the CDS premium  $P$  should equal the credit spread  $S$ . Based on this argumentation the so called basis,  $P - S$ , consequently equals zero. As in the market a basis different from zero can be observed, various factors causing the divergence between CDS spread and credit spread need to be considered. A detailed examination of this question can be found in a publication by Lehman Brothers (0).

As it is probably the most important factor affecting the basis, only the counterparty default risk should be mentioned explicitly. Unlike in bond markets, where the transaction between the issuer and the investor does not involve

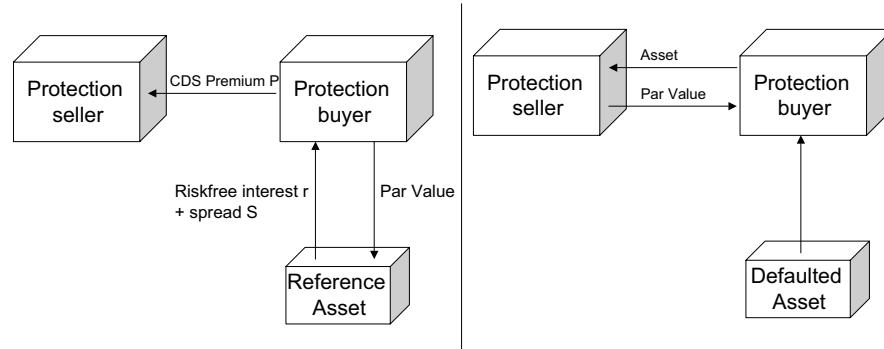


Figure 13: Payment structure of a CDS before and in the event of default

any other credit risk factors apart from the obligor's default risk, a new dimension of credit risk is implied on the CDS market. The derivative contract is established with a protection seller, who himself can default. The question of default correlation between the issuer of the underlying of the CDS and the protection seller needs to be considered. Generally the concern is on the side of the protection buyer, who will demand a reduced CDS premium for taking the additional risk of default of the protection seller. The result will be a decrease in the basis.

Based on a variety of possible influence factors it can not be concluded, that a strictly positive or negative basis should be observed in the market. The examination of the basis, of observable maturity effects in the CDS market and the comparison the results obtained from bond credit spreads are the topics of the following chapters.

## 4.2 Empirical Results from the CDS Market

### 4.2.1 Description of the Sample

The data source for the CDS sample is a database, where Credit Default Swap indicators are collected. These indicators are bid and ask quotes provided by various market participants and do not necessarily represent real trades. In the further discussion the arithmetic mean between bid and ask, called the mid quote, is examined. The maturity of the contract, the underlying company, the credit rating, currency and various other static information completes the sample. The time horizon between January 2001 and March 2004 is covered, where in the beginning of the sample period only about 200, later up to 3000 quotes per trading day are available. The major part of the sample are single name corporate CDS quotes denominated in U.S.-\$ and consequently the analysis has been restricted solely to those contracts. Credit Default Swaps are OTC-contracts usually equipped with integer maturities, where the 5-year CDS is by far the most common specification, followed by maturities of 1, 3, 7

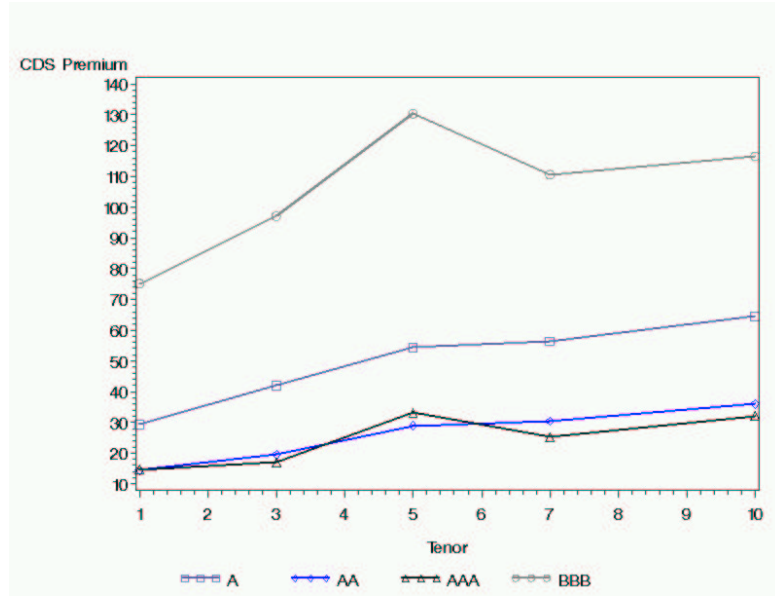


Figure 14: Investment-grade CDS-quotes averaged across sample period

and 10 years.

#### 4.2.2 Results across Time

In a first step the behavior of the CDS-quotes over the whole sample period should be examined. Figures 14 and 15 show the average mid-quotes of the whole sample for different maturities, classified according to the credit rating. Note that all quotes during the following analysis are given in basispoints, following the market convention for the quotation of the CDS premium. For the investment-grade rating classes a clear positive relationship between CDS-quote and maturity is observable. Furthermore it is remarkable, that the quotes for AAA- and AA-rated reference entities are very close, where for the 5-year maturity AAA- even exceed the AA-quotes. Apart from this anomaly, the 5-year quotes exhibit a noticeable behavior during the whole analysis. Representing the point with the highest liquidity, they constitute a peak in the run of the curve for many rating classes. Although there appear to be no definite explanations for this behavior, it is necessary to point this effect out, in particular as it becomes more evident during the analysis of single trading days. Regarding the subinvestment rating classes, especially for rating class B, a hump-shaped behavior can be observed.

In table 7 mean, standard-deviation and coefficient of variation are presented for the quotes, covering the whole time horizon. This illustrates the extraordinary variability of the underlying pool of data. The coefficient of variation is generally even higher than for the bond sample. One aspect explaining the high



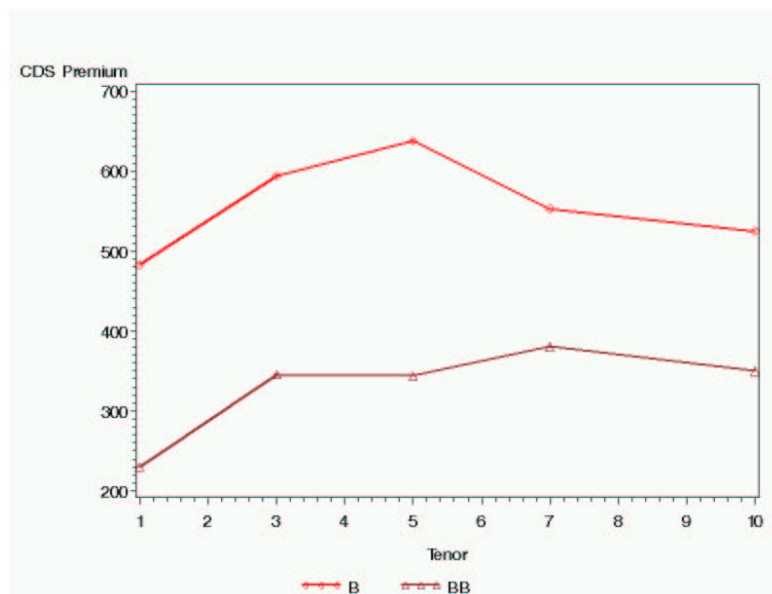


Figure 15: Subinvestment-grade CDS-quotes averaged across sample period

variability deals with the speed of reaction on the different financial markets. Hull, Predescu and White (0) examine, whether the CDS market anticipates the change of credit ratings. Especially for rating downgrades they conclude, that the CDS-market perceps this negative event well in advance, observable by a significant increase of the CDS-premium up to 90 days prior to the rating event. Longstaff, Mithal and Neis (0) apply a vector-autoregression model to analyze the lead-lag relations between the CDS-, bond- and stockmarket. Although limited by a large number of exceptions they generally observe, that changes in CDS-premia lead changes in corporate bond yields, indicating that the CDS-market incorporates credit-relevant information more quickly. Considering these findings it is imaginable, that the quotes in a specific rating class can cover a wider range, because participants at the CDS-markets estimate the credit quality of some issuers within this class to be significantly better or lower, although this has not yet been reflected in a change of the credit rating. This is only presented as one possible explanation, as it is not possible to test this hypothesis with the available data.

#### 4.2.3 Results for February 11th, 2004

Results from the examination of subsets from the pool of CDS-quotes for single trading days are presented in the following. For the same reference date as in the analysis of the bond spreads, February 11th, 2004, the mid-quotes of all available CDS-contracts are examined. By averaging those quotes for the maturities of 1, 3, 5, 7 and 10 years, figures 16 and 17 are obtained.

	Time Horizon [years]				
	1	3	5	7	10
	AAA				
$\mu$	14.69	17.09	33.1	25.2	32.11
$\sigma$	9.87	14.01	24.91	15.27	17.02
$\sigma/\mu$	0.67	0.82	0.75	0.61	0.53
n	1463	4199	11866	3685	3274
	AA				
$\mu$	14.62	19.55	28.98	30.49	36.13
$\sigma$	48.49	16.61	23.68	18.13	21.53
$\sigma/\mu$	3.32	0.85	0.82	0.59	0.6
n	1721	23143	37491	20749	22149
	A				
$\mu$	29.51	42.01	54.44	56.46	64.51
$\sigma$	23.92	31.18	47.75	34.1	35.99
$\sigma/\mu$	0.81	0.74	0.88	0.6	0.56
n	14389	57081	138746	46091	46858
	BBB				
$\mu$	75.17	97.13	130.45	110.62	116.59
$\sigma$	74.74	84.38	131.36	77.05	71.2
$\sigma/\mu$	0.99	0.87	1.01	0.7	0.61
n	19037	47619	164328	39660	41245
	BB				
$\mu$	230.05	345.71	344.2	380.79	350.05
$\sigma$	193.33	364.73	248.8	306.4	270.42
$\sigma/\mu$	0.84	1.06	0.72	0.8	0.77
n	1584	7542	41114	4608	5229
	B				
$\mu$	482.88	594.14	638.05	552.93	524.9
$\sigma$	417.03	471.86	695.41	377.75	328.28
$\sigma/\mu$	0.86	0.79	1.09	0.68	0.63
n	24	979	8208	687	688

Table 7: Statistical indicators for CDS-quotes across time horizon

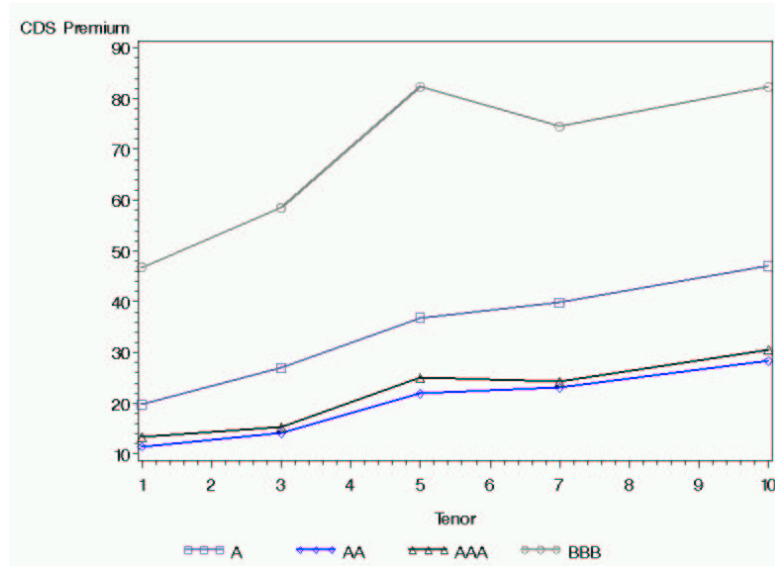


Figure 16: Average investment-grade CDS-quotes as of February 11th, 2004

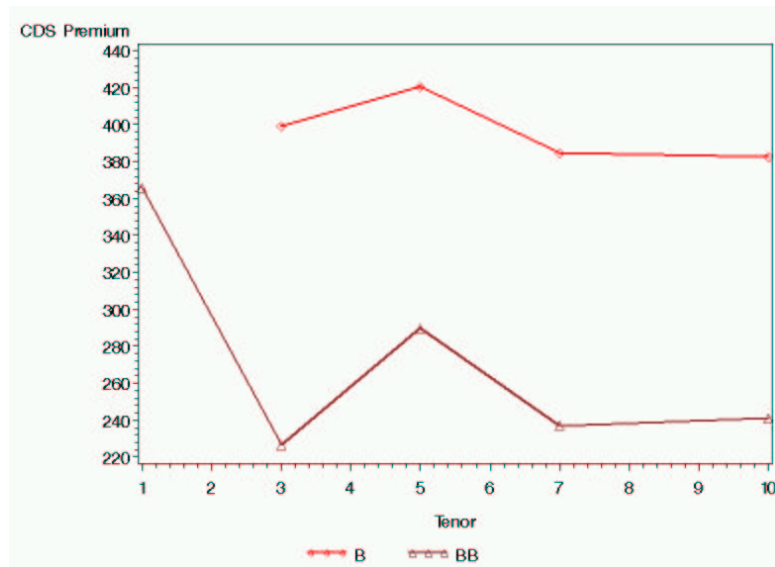


Figure 17: Average subinvestment-grade CDS-quotes as of February 11th, 2004

	n	$\rho$	b	t-stat.	p-value	R <sup>2</sup>
AAA	86	0.391	1.882	3.89	0	0.153
AA	212	0.41	1.819	6.51	<0.0001	0.168
A	915	0.444	2.878	14.96	<0.0001	0.197
BBB	1314	0.166	3.253	6.1	<0.0001	0.028
BB	258	-0.125	-12.101	-2.02	0.044	0.016
B	41	-0.033	-6.166	-0.2	0.839	0.001

Table 8: Correlation and Regression results for CDS-quotes as of February 11th, 2004

	n	$\rho$	b	t-stat.	p-value	R <sup>2</sup>
AAA	5	0.982	3.032	8.97	0.003	0.964
AA	5	0.976	1.923	7.71	0.005	0.953
A	5	0.951	1.958	5.35	0.013	0.904
BBB	5	0.853	3.842	2.84	0.066	0.728
BB	5	-0.63	-10.409	-1.4	0.255	0.397
B	4	-0.634	-3.727	-1.16	0.366	0.402

Table 9: Correlation and Regression results for average CDS-quotes as of February 11th, 2004

A positive relationship between CDS-quotes and maturity is clearly observable for the investment-grade classes. Again the peak for the 5-year maturity can be identified in particular for AAA and BBB rating classes. Although unexplainable it has to be noted, that the quotes for the AAA rating class are uniformly higher than for AA. Regarding the subinvestment-grade rating classes no systematic effect for this date can be identified. For BB the 1-year quote represents an outlier compared to the examination covering the whole sample period. Interpretation of the depiction for rating class B is questionable, due to the fact that the quotes for maturities of 3, 7 and 10 years are only based on two data points each. The findings are supported by the correlation coefficient and the results of a linear regression model, as presented in table 8 and 9. Although a positive correlation and slope coefficient for the investment-grade CDS-quotes is obtained while carrying out the regression on the raw data, the positive spread-maturity relationship is in particular supported by analyzing the average CDS-quotes, where R<sup>2</sup> reaches values between 72% and 96%.

Comparable to the proceeding while analyzing bondspreads, spreads from forward rates are calculated for the maturities corresponding with the CDS-quotes. As all CDS-quotes in the sample are written on senior unsecured underlyings, the value-weighted average recovery rate of  $\mu=0.44$  for senior unsecured issues in 2003 as reported by Moody's (0) is used in calculation. The calculated spreads are given in table 10 and a graphical representation can be

	Time Horizon [years]				
Rating	1	3	5	7	10
AAA	0	0.7	1.3	2.3	3.1
AA	0.6	2	3.5	5	5.7
A	2.8	5.5	8	9.4	10.5
BBB	20.6	32.8	40.8	41	37
BB	80.9	148.2	157	150.8	126.1
B	373.3	432.3	373.1	309.6	227.4
CCC	2105	1047.1	736.6	504.3	324.3

Table 10: Calculated spreads [bp], based on forward-pd's, for February 11th, 2004

found in figure 18 and 19. Again the calculated spreads are uniformly below the observed CDS-quotes, in particular in the investment-grade rating classes. The results of a regression for the market spreads with the calculated spreads as explanatory variable are given in table 11. Like in the bond sample the fit is in particular good for the investment-grade rating classes and the positive intercept and slope coefficient reflects the above mentioned observation of market spreads being higher than calculated spreads. The negative correlation and slope coefficient for rating class BB contradicts the expectation and the quality of the fit of the regression declines while moving to rating class B.

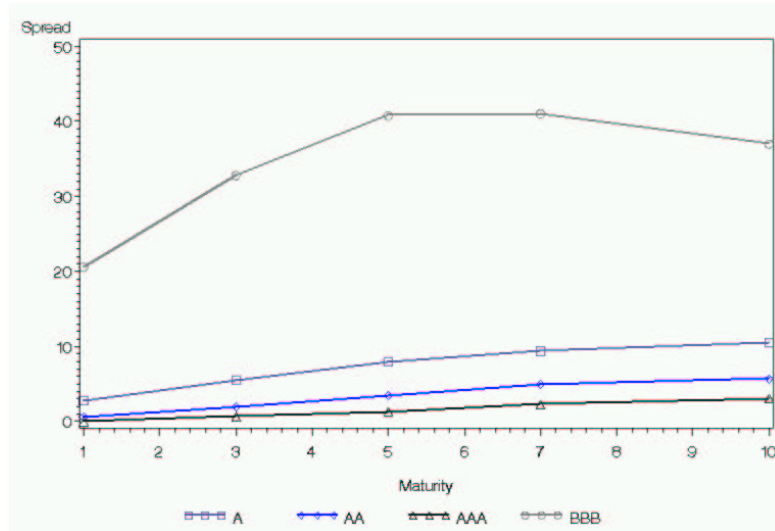


Figure 18: Calculated spreads for February 11th, 2004

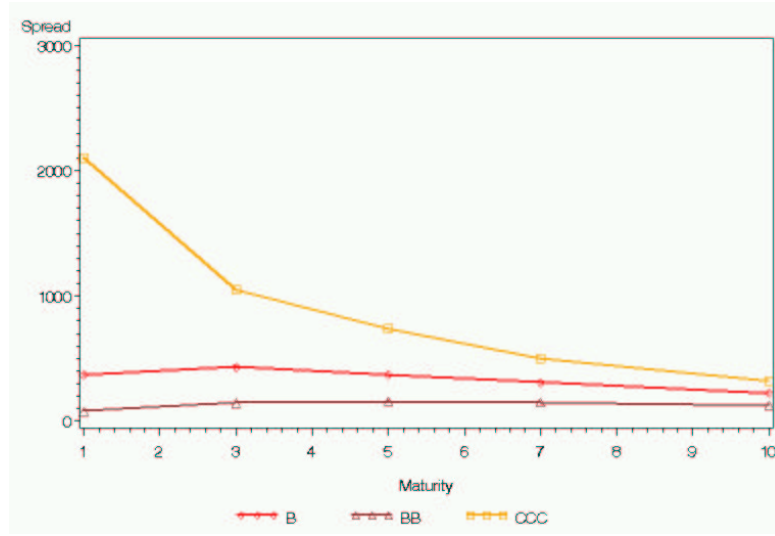


Figure 19: Calculated spreads for February 11th, 2004

	n	$\rho$	a	t-stat	p-value	b	t-stat	p-value	R <sup>2</sup>
AAA	5	0.927	13.653	5.82	0.01	5.416	4.27	0.024	0.859
AA	5	0.972	9.184	5.43	0.012	3.154	7.21	0.006	0.946
A	5	0.991	9.351	4.61	0.019	3.423	13.09	0.001	0.983
BBB	5	0.905	10.881	0.68	0.548	1.684	3.69	0.035	0.819
BB	5	-0.772	461.863	5.01	0.015	-1.432	5.01	0.015	0.596
B	4	0.642	353.57	9.48	0.011	0.128	1.18	0.358	0.412

Table 11: Regression of CDS market spreads by calculated spreads for February 11th, 2004

#### 4.2.4 Results for August 11th, 2003

Like for the bond sample the analysis of CDS-quotes has been conducted for the second reference date, August 11th, 2003. The results are presented in appendix C. They are a confirmation of all the conclusions drawn above.

- The positive relationship between CDS-quote and maturity is clearly observable for the investment-grade rating classes.
- Inconsistent behavior like the peak for the 5-year-quote and the fact, that AAA-quotes are higher than AA-quotes, recurs for this reference date.
- The lack of data quantity for rating class B prevents conclusions regarding a maturity effect and also for BB the resulting graph is of limited explanatory power, as especially the 1-year quote is heavily influenced by a small number of observations.

- The CDS-quotes are uniformly higher for this reference date, which corresponds to the results derived from the analysis of bond spreads.

#### 4.2.5 Comparison of the Results for Bond and CDS-market

As pointed out in 4.1.1, the spread of the underlying asset is a first estimation for the CDS-quote. Although the bond sample does not necessarily consist of the underlying assets of the CDS-sample, a comparison on an aggregated level should be conducted. At least for the investment-grade rating classes some interesting results can be obtained from the comparison of CDS-quotes and bond spreads for the same trading day. The positive relationship between CDS-quote and maturity is much more obvious than in the bond sample. As mentioned before, the pure default risk is reflected in a CDS more precisely than it is captured in the bond spread. The fact that the positive slope for the CDS-quotes is more apparent supports the argumentation, which suggests the possible deterioration in credit quality of the issuer as a reason for increasing credit spreads.

The second remarkable result of the comparison deals with the basis. The basis has been introduced as the difference of CDS-quote and bond spread. Various arguments have been provided to explain a basis different from zero. In the present data sample the CDS-quotes are almost uniformly lower than the bond spreads, resulting in a negative basis. For longer maturities the basis decreases, as the maturity effect causes CDS-quotes to increase, whereas this effect is less strong for the bond spreads.

Due to the variability in the bond spreads, the inhomogeneous results for the CDS-quotes and the general lack of data, reliable conclusions can not be drawn for the subinvestment-grade rating classes.

## 5 Conclusion

Based on the “crisis-at-maturity-hypothesis” and subsequent argumentations, a split behavior of bond spreads with respect to maturity is presumed. This is supported by the theoretical predictions of credit risk models, following both the structural and the reduced-form approach. Empirical research regarding these maturity effects of bond spreads has been very limited so far. Furthermore the results of the few available studies vary and the answer to the question if the predicted maturity effects are observable in the market, is still controversial. After having summarized the former empirical work and the consideration of possible additional factors contributing to the credit spread, the results of an analysis covering about 2000 -denominated bonds are presented. Although dominated by the extraordinary variability of spread data even within the same rating class, a positive relationship between spread and maturity was detected for the investment-grade rating classes, whereas the results for the subinvestment-grade issues are only of limited explanatory power due to a lack in data quantity. However, during the whole analysis no evidence for a negative relationship was

found.

Comparable results were obtained from the examination of a sample of credit default swap quotes. A credit default swap is a derivative financial contract, where the protection seller has to pay if and only if a default event of the reference issue occurs. This specification extracts the default risk of the reference issue most precisely and therefore the presumed maturity effects on the bond market should be observable in the credit default swap market as well. They should even appear more clearly, as many of the potential factors affecting bond spreads do not affect the credit default swaps. While trying to analyze CDS-quotes another problem appears. As these derivative instruments are OTC-contracts, reliable and exhaustive price information is not easily available. An internal database of so-called price indicators (bid and ask quotes provided by different market participants) was used for examination. Generally the positive relationship between spread and maturity was essentially apparent. Furthermore it was possible to compare bond-spreads and CDS-quotes regarding the overall level and thus obtain evidence regarding the so-called basis, the difference between bond spreads and CDS-premium. For the CDS-quotes of subinvestment-grade issues, the same limitations as for the bond spreads hold.

## A Stable Effects in the Distribution of Regression Residuals

While interpreting regression results and the  $R^2$ -statistic as a measure of fit, the distribution of the residuals, that is the difference between observed and predicted values, has to be considered. In the standard setting of a regression model, the residuals are assumed to be normally distributed. If this is not the case,  $R^2$  might not reflect the fit of the regression appropriately. A large number of residuals of small and several residuals of high absolute value suggest a heavy-tailed distribution. The  $\alpha$ -stable distributions constitute a class of distributions allowing the modelling of skewness and heavy tails. They are characterized by the parameter vector  $\Theta = (\alpha, \beta, \sigma, \mu)$ , with  $\alpha \in (0, 2]$  being the index of stability, the skewness parameter  $\beta \in [-1, 1]$ , the scale parameter  $\sigma \in \mathcal{R}_+$  and the location parameter  $\mu \in \mathcal{R}$ . For  $\alpha < 2$  the distributions exhibit peakedness and heavy tails. The normal distribution  $\mathcal{N}(\mu, \sigma^2)$  belongs to the stable distributions as well and is obtained for the parameter choice  $\Theta = (2, 0, \frac{\sigma}{\sqrt{2}}, \mu)$ . Currently a large amount of research is carried out in the field of stable distributions. One example for advanced literature dealing with stable distributions and their applications to finance is Rachev, Mittnik (0).

The regression residuals of bond market spreads on maturity for February 11th, 2004 are used to estimate the parameters of an  $\alpha$ -stable distribution. The results are given in table 12 and figures 20 to 24 illustrate the fact, that the distribution of the residuals matches more closely the  $\alpha$ -stable than the normal distribution.



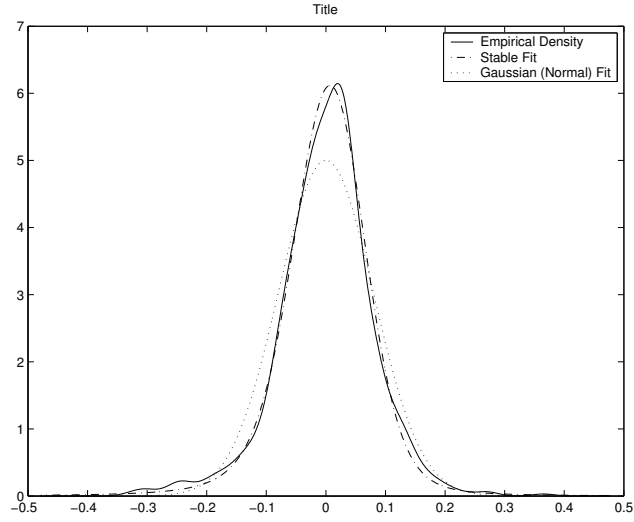


Figure 20:  $\alpha$ -stable fit of the regression residuals for rating AAA

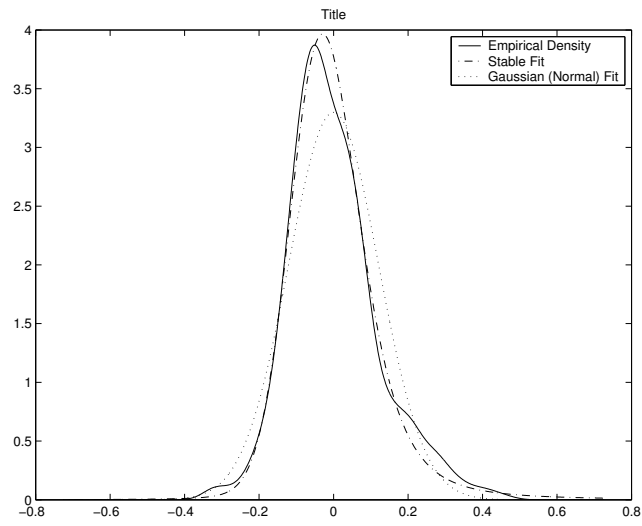


Figure 21:  $\alpha$ -stable fit of the regression residuals for rating AA

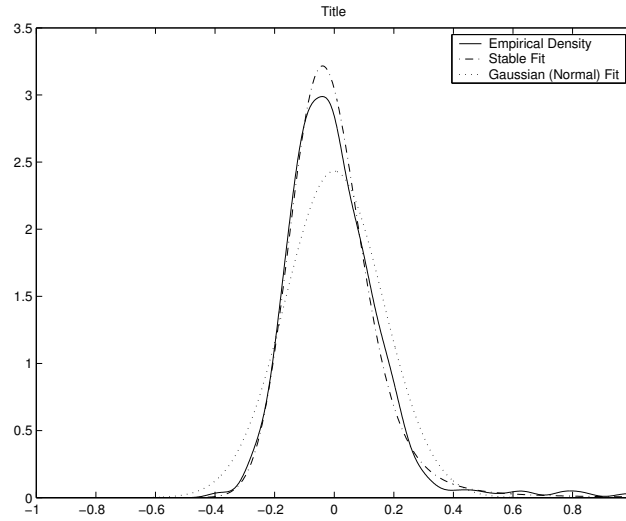


Figure 22:  $\alpha$ -stable fit of the regression residuals for rating A

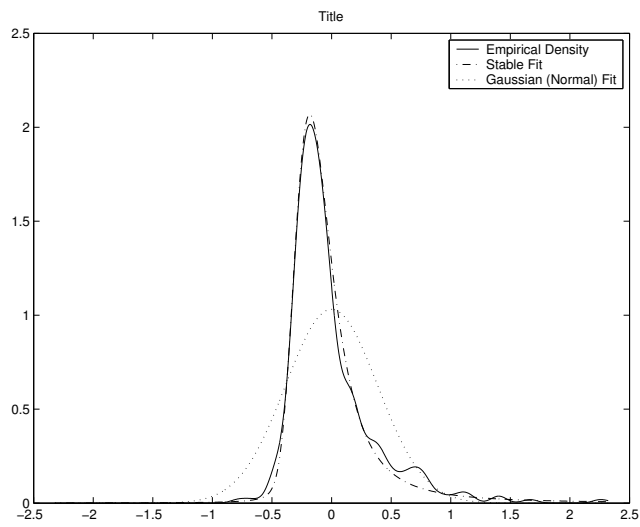


Figure 23:  $\alpha$ -stable fit of the regression residuals for rating BBB

Rating	Time Horizon [years]				
	n	$\alpha$	$\beta$	$\sigma$	$\mu$
AAA	707	1.7061	-0.2790	0.0464	-0.0006
AA	426	1.6655	0.7874	0.0712	0.0087
A	595	1.7412	0.9755	0.0869	0.0032
BBB	334	1.3084	0.8894	0.1377	0.0861
BB	46	1.6243	1.0000	0.5804	0.1366

Table 12: Parameters of an  $\alpha$ -stable distribution for regression residuals of bondspreads

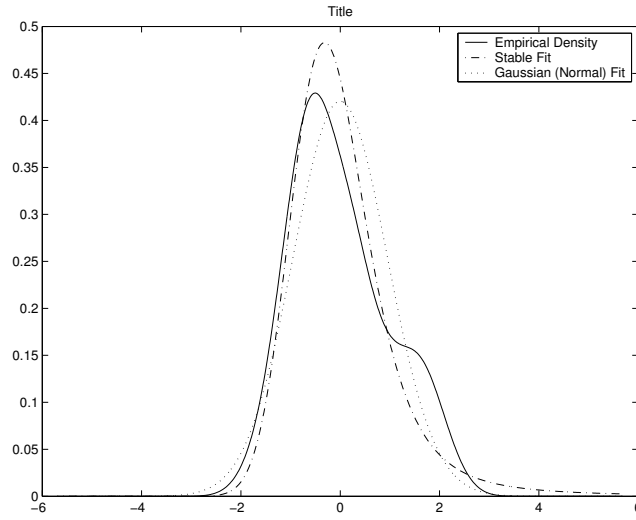


Figure 24:  $\alpha$ -stable fit of the regression residuals for rating BB

## B Bond Data for August 11th, 2003

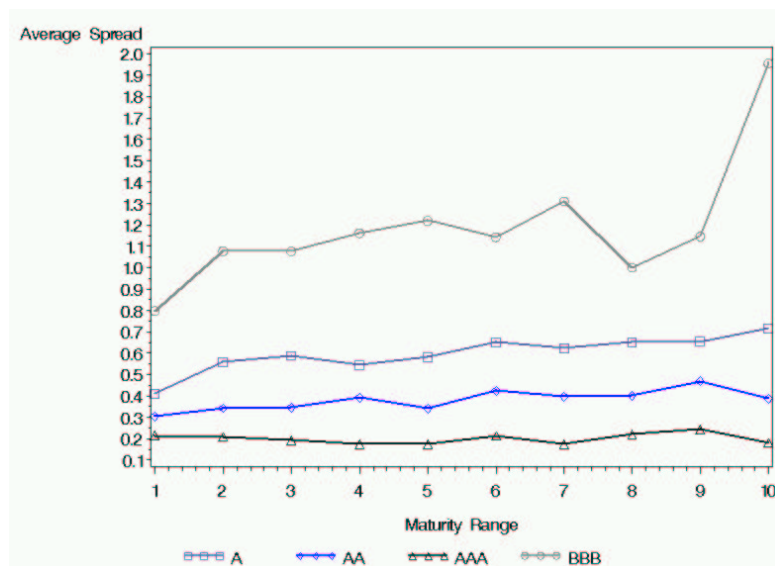


Figure 25: Average bondsreads as of August 11th, 2003 for investment-grade rated issues

	n	$\rho$	b	t-stat.	p-value	R <sup>2</sup>
AAA	689	-0.017	-0.001	-0.46	0.649	0.0003
AA	369	0.176	0.012	3.42	0.0007	0.031
A	538	0.254	0.024	6.07	< 0.0001	0.064
BBB	271	0.237	0.059	4.01	< 0.0001	0.056
BB	46	0.203	0.123	1.37	0.1762	0.041
B	38	0.556	0.484	4.02	0.0003	0.310

Table 13: Correlation and Regression results for bondsreads as of August 11th, 2003

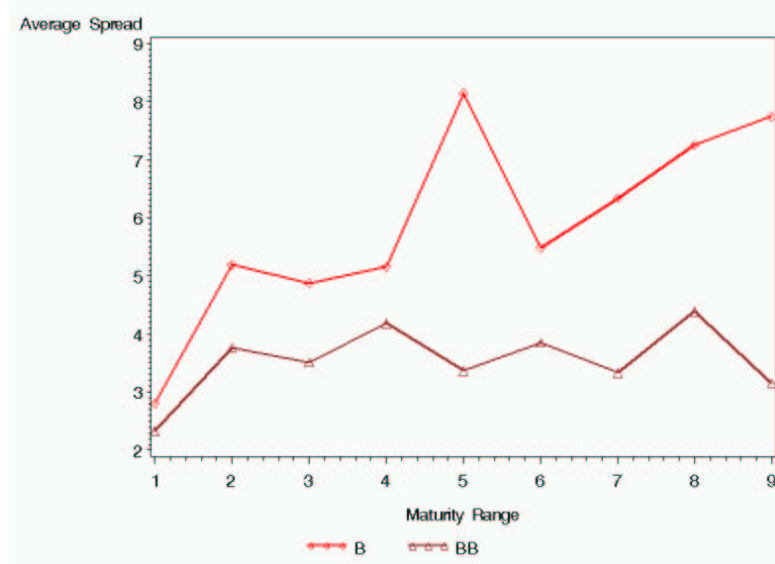


Figure 26: Average bondspreads as of August 11th, 2003 for subinvestment-grade rated issues

	n	$\rho$	b	t-stat.	p-value	R <sup>2</sup>
AAA	10	0.183	0.001	0.53	0.614	0.033
AA	10	0.755	0.011	3.26	0.012	0.57
A	10	0.87	0.022	5	0.001	0.757
BBB	10	0.661	0.066	2.49	0.037	0.437
BB	9	0.334	0.075	0.94	0.379	0.112
B	9	0.795	0.487	3.5	0.01	0.632

Table 14: Correlation and Regression results for maturity buckets of bondspreads as of August 11th, 2003

	Time Horizon [years]									
	1	2	3	4	5	6	7	8	9	10
	AAA									
$\mu$	0.2	0.2	0.18	0.17	0.17	0.2	0.17	0.22	0.24	0.18
$\sigma$	0.08	0.08	0.1	0.07	0.09	0.11	0.07	0.09	0.07	0.06
$\sigma/\mu$	0.4	0.39	0.54	0.42	0.52	0.54	0.44	0.39	0.31	0.32
n	121	121	96	83	85	47	50	27	30	29
	AA									
$\mu$	0.29	0.32	0.34	0.39	0.32	0.39	0.38	0.38	0.45	0.37
$\sigma$	0.12	0.13	0.2	0.21	0.16	0.15	0.22	0.14	0.24	0.12
$\sigma/\mu$	0.41	0.41	0.58	0.54	0.5	0.39	0.59	0.38	0.54	0.32
n	45	54	43	50	49	46	25	24	16	17
	A									
$\mu$	0.41	0.55	0.58	0.53	0.57	0.63	0.62	0.63	0.63	0.68
$\sigma$	0.18	0.21	0.3	0.16	0.26	0.19	0.26	0.21	0.2	0.33
$\sigma/\mu$	0.45	0.39	0.52	0.31	0.45	0.3	0.42	0.33	0.31	0.48
n	68	62	85	57	77	65	43	28	29	24
	BBB									
$\mu$	0.8	1.08	1.08	1.16	1.22	1.14	1.31	1	1.15	1.96
$\sigma$	0.4	0.54	0.6	0.56	0.67	0.45	0.74	0.22	0.42	0.58
$\sigma/\mu$	0.5	0.5	0.55	0.48	0.55	0.39	0.56	0.22	0.37	0.3
n	33	31	50	32	43	29	21	14	10	8
	BB									
$\mu$	2.33	3.77	3.51	4.19	3.37	3.85	3.33	4.4	3.15	–
$\sigma$	1.44	1.81	1.14	1.6	1.27	0	0.51	1.3	1.26	–
$\sigma/\mu$	0.62	0.48	0.32	0.38	0.38	0	0.15	0.3	0.4	–
n	10	7	9	7	4	1	3	2	3	–
	B									
$\mu$	2.8	5.19	4.88	5.15	8.15	5.48	6.34	7.26	7.75	–
$\sigma$	1	2.01	1.2	1.32	0.87	4.01	1.15	1.39	0	–
$\sigma/\mu$	0.36	0.39	0.25	0.26	0.11	0.73	0.18	0.19	0	–
n	6	11	4	6	2	2	3	3	1	–

Table 15: Statistical indicators for bondspreads as of August 11th, 2003

	Time Horizon [years]									
Rating	1	2	3	4	5	6	7	8	9	10
AAA	0	0	0.007	0.009	0.012	0.017	0.022	0.03	0.029	0.028
AA	0.005	0.01	0.019	0.025	0.033	0.039	0.047	0.05	0.051	0.052
A	0.026	0.039	0.051	0.062	0.074	0.081	0.087	0.089	0.094	0.096
BBB	0.192	0.273	0.304	0.355	0.377	0.387	0.377	0.362	0.344	0.334
BB	0.754	1.137	1.374	1.526	1.446	1.443	1.377	1.297	1.233	1.129
B	3.473	4.003	3.989	3.752	3.412	3.09	2.799	2.548	2.262	2.004
CCC	19.461	12.648	9.557	7.707	6.642	5.501	4.48	3.676	3.241	2.787

Table 16: Calculated spreads [%], based on forward-pd's, for August 11th, 2003

	n	$\rho$	a	t-stat	p-value	b	t-stat	p-value	$R^2$
AAA	10	0.282	0.184	13.73	<.0001	0.594	0.83	0.429	0.08
AA	10	0.762	0.298	13.43	<.0001	2.002	3.33	0.01	0.581
A	10	0.89	0.388	10.46	<.0001	2.79	5.53	0.001	0.793
BBB	10	-0.445	1.417	7.62	<.0001	-0.538	-1.41	0.197	0.198
BB	9	0.701	1.174	1.27	0.246	1.843	2.6	0.036	0.491
B	9	-0.584	10.977	4.04	0.005	-1.561	-1.9	0.099	0.341

Table 17: Regression of bond market spreads by calculated spreads for August 11th, 2003

## C CDS Data for August 11th, 2003

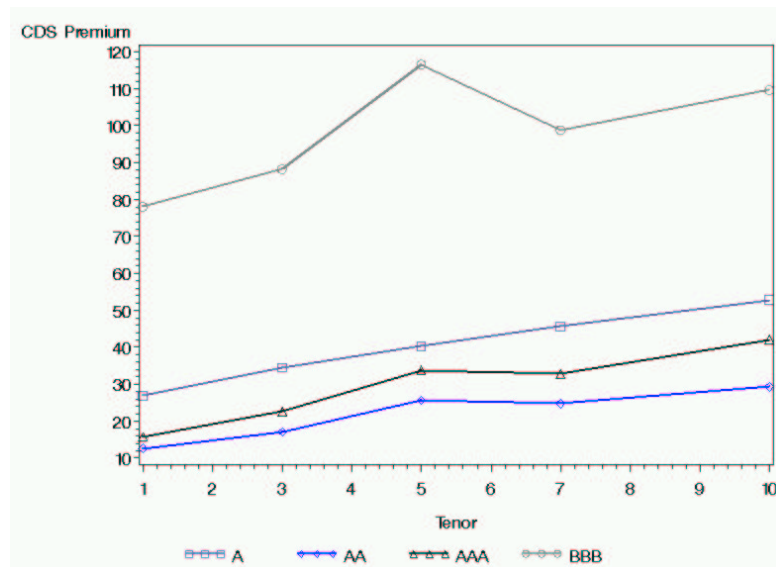


Figure 27: Average investment-grade CDS-quotes as of August 11th, 2003

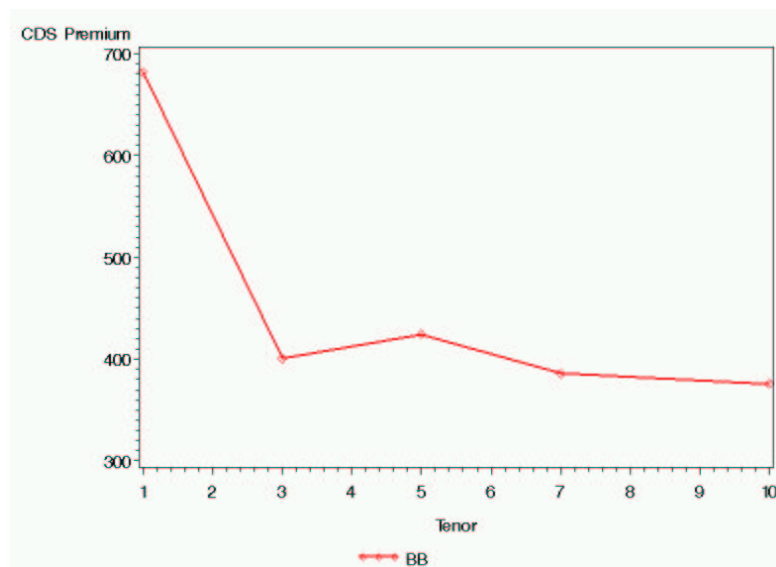


Figure 28: Average subinvestment-grade CDS-quotes as of August 11th, 2003



	n	$\rho$	b	t-stat.	p-value	R <sup>2</sup>
AAA	75	0.413	2.759	3.87	0	0.171
AA	179	0.317	1.566	4.45	<0.0001	0.1
A	744	0.351	2.776	10.2	<0.0001	0.123
BBB	873	0.085	2.873	2.53	0.012	0.007
BB	160	-0.2	-31.209	-2.56	0.011	0.04

Table 18: Correlation and Regression results for CDS-quotes as of August 11th, 2003

	n	$\rho$	b	t-stat.	p-value	R <sup>2</sup>
AAA	5	0.963	2.837	6.15	0.009	0.927
AA	5	0.937	1.835	4.67	0.019	0.878
A	5	0.995	2.834	18.01	0.0004	0.99
BBB	5	0.729	3.257	1.85	0.162	0.531
BB	5	-0.74	-27.315	-1.9	0.153	0.548

Table 19: Correlation and Regression results for average CDS-quotes as of August 11th, 2003

	Time Horizon [years]				
Rating	1	3	5	7	10
AAA	0.0	0.7	1.3	2.3	3.1
AA	0.6	2.0	3.5	5.1	5.7
A	2.8	5.5	8.0	9.5	10.5
BBB	20.6	32.8	40.9	41.2	37.3
BB	80.9	148.2	157.3	151.4	127.1
B	373.3	432.2	373.9	311.2	229.8
CCC	2105.0	1046.6	738.3	507.8	329.1

Table 20: Calculated spreads [bp], based on forward-pd's, for August 11th, 2003

	n	$\rho$	a	t-stat	p-value	b	t-stat	p-value	R <sup>2</sup>
AAA	5	0.943	17.833	6.05	0.009	7.846	4.92	0.016	0.89
AA	5	0.949	11.586	5.1	0.015	3.06	5.22	0.014	0.901
A	5	0.983	17.384	6.54	0.007	3.127	9.15	0.003	0.965
BBB	5	0.849	44.549	2.25	0.11	1.555	2.78	0.069	0.72
BB	5	-0.881	937.26	6.12	0.009	-3.636	-3.23	0.048	0.776

Table 21: Regression of CDS market spreads by calculated spreads for August 11th, 2003

## References

- BASEL COMMITTEE ON BANKING SUPERVISION : International Convergence of Capital Measurement and Capital Standards: A Revised Framework, *Bank for International Settlements*, June 2004, available at <http://www.bis.org/publ/bcbs107.htm>
- BLACK, F. AND SCHOLES, M. : The Valuation of Options and Corporate Liabilities, *Journal of Political Economy*, 81 (3), pp. 637–654
- BOHN, J.R. : Characterizing Credit Spreads, *University of California, Berkeley, Working Paper*, June 1999
- DELIANEDIS, G. AND GESKE, R. : The Components of Corporate Credit Spreads: Default, Recovery, Tax, Jumps, Liquidity and Market Factors, *Anderson Graduate School of Management, University of California, Los Angeles*, 2001, Finance Paper 22.01
- ELTON, E.J., GRUBER, M.J., AGRAWAL, D. AND MANN, C. : Explaining the Rate Spread on Corporate Bonds, *Journal of Finance*, 56 (1), 2001, pp. 247–277
- FONS, J.S. : Using Default Rates to Model the Term Structure of Credit Risk, *Financial Analysts Journal*, Sep–Oct 1994, pp. 25–33
- HELWEGE, J. AND TURNER, C.M. : The Slope of the Credit Yield Curve for Speculative–Grade Issuers, *Journal of Finance*, 54 (5), 1999, pp. 1869–1884
- HULL, J.; PREDESCU, M.; WHITE, A. : The Relationship between Credit Default Swap Spreads, Bond Yields and Credit Rating Announcements, *Forthcoming, Journal of Banking and Finance*, 2004, available at: <http://www.rotman.utoronto.ca/~hull/DownloadablePublications/>, date of last access: July 5th, 2004
- JARROW, R.A., LANDO, D. AND TURNBULL, S.M. : A Markov Model for the Term Structure of Credit Risk Spreads, *The Review of Financial Studies*, 10 (2), 1997, pp. 481–523
- JOHNSON, R.E. : The Term Structure of Corporate Bond Yields as a Function of Risk of Default, *Journal of Finance*, 22 (2), 1967, pp. 313–345
- JPMORGAN : Euro Credit Daily Analytics, *Credit and Index Strategy, J.P.Morgan Securities Ltd.*, February 12th, 2004, London
- LEE, C. : The Pricing of Corporate Debt – A Note, *Journal of Finance*, 36 (5), 1981, pp 1187–1189
- LEHMAN BROTHERS : Explaining the Basis: Cash versus Default Swaps, *Structured Credit Research*, May 2001
- LONGSTAFF, F.A.; MITHAL, S.; NEIS, E. : The Credit Default Swap Market: Is Credit Protection Priced Correctly?, *Working Paper*, 2003, available at <http://www.mcombs.utexas.edu/dept/finance/seminars/2003/cdsnew.pdf>, date of last access: July 5th, 2004
- MERTON, R.C. : On the Pricing of Corporate Debt: The risk structure of interest rates, *Journal of Finance*, 2 (2), 1974, pp. 449–470

- MOODY'S INVESTOR SERVICE, INC. : Default & Recovery Rates of Corporate Bond Issuers, *Moody's Special Comment*, Report Number: 80989, 2004
- PITTS, C.G. AND SELBY, M.J. : The Pricing of Corporate Debt: A Further Note, *Journal of Finance*, 38 (4), pp. 1311–1313
- RACHEV, S.T AND MITTNIK, S. : *Stable Paretian Models in Finance*, 2000, John Wiley & Sons, Inc.
- SARIG, O. AND WARGA, A. : Some Empirical Estimates of the Risk Structure of Interest Rates, *Journal of Finance*, 44 (5), 1989, pp. 1351–1360
- SAUNDERS, A. : *Credit Risk Measurement*, 1999, John Wiley & Sons, Inc.
- SCHÖNBUCHER, P.J. : *Credit Derivatives Pricing Models*, 2003, John Wiley & Sons, Inc.
- STANDARD & POOR'S : Corporate Default in 2003 Recede From Recent Highs, *Standard & Poor's Ratingsdirect Research*, 2004
- UHRIG-HOMBURG, M. : Valuation of Defaultable Claims – A Survey, *Schmalenbach Business Review*, Vol. 54, 2002, pp. 24–57