Pricing of Credit Default Index Swap Tranches with One-Factor Heavy-Tailed Copula Models

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Abstract

In this paper, we provide two one-factor heavy-tailed copula models for pricing a collateralized debt obligation and credit default index swap tranches: (1) a one-factor double t distribution with fractional degrees of freedom copula model and (2) a one-factor double mixture distribution of t and Gaussian distribution copula model. A tail-fatness parameter is introduced in each model, allowing one to change the tail-fatness of the copula function continuously. Fitting our model to comprehensive market data, we find that a model with fixed tail-fatness cannot fit market data well over time. The two models that we propose are capable of fitting market data well over time when using a proper tail-fatness parameter. Moreover, we find that the tail-fatness parameters change dramatically over a one-year period.

JEL Classification: G12, G13.

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1 Introduction

Credit derivatives permit market participants to transfer credit risk for individual credits and a portfolio of credits. The major type of credit derivative is the credit default swap, which includes single-name swaps, basket swaps, credit default index swaps, and credit default index swap tranches. While several one-factor and two-factor models have been proposed in the literature for single-name credit default swaps, in the case of a portfolio of credits, the prevailing research is not as well developed; moreover, empirically not much work has been done in developing models that fit market data. The difficulty in modeling lies in estimating the correlation risk for a portfolio of credits. As Duffie (2004) commented: "Banks, insurance companies and other financial institutions managing portfolios of credit risk need an integrated model, one that reflects correlations in default and changes in market spreads. Yet no such model exists." Amato and Gyntelberg (2005, p. 74) of the Bank for International Settlements noted that while a few models have been proposed, the modeling of these correlations is "complex and not yet fully developed."

Several models have been proposed for pricing tranches of credit default index swaps traded in North America and Europe. While the developers of these models offer empirical support, the time period covered in these studies is usually only one trading day. Thus, a fair conclusion is that empirical evidence regarding how these models fit market data is scant. This is due to limited market data. The conclusions about the validity of the proposed models are therefore time-period or market condition dependent.

The purpose of this paper is twofold. First, we provide two extensions to the proposed one-factor Gaussian copula model that have been suggested in the literature. The advantage of our two models is that they include a parameter that controls for the tail-fatness of the copula function. Second, we provide a far more extensive empirical analysis than prior studies, using market data from September 1, 2004 to August 31, 2005 (a total of 248 trading days for the 5-year index and 235 trading days for the 10-year index). Moreover, we compare one of our proposed models in this paper with the one-factor double t copula model by Hull and White (2004).

Our presentation in this paper is as follows. In the next section, we provide a brief review of credit default index swap tranches. In Section 3, we review one-factor copula function models that have been proposed in the literature. The two models we propose — a one-factor double t distribution with factional degrees of freedom copula model and a one-factor double mixture distribution of t and Gaussian distribution copula model — are explained in Section 4. The two models fall into the general category of heavy-tailed copula models. Implementation of the two models is explained in Section 5. The market data analyzed and the empirical results are contained in Section 6. Our conclusions are given in the last section.

2 Credit Default Index Swap Tranches

In a credit default swap (CDS), the protection buyer pays a fee, the swap premium, to the protection seller in exchange for the right to receive a payment conditional upon the occurrence of a "credit event" with respect to a reference entity or reference entities for which credit protection is being sold. In the trade's documentation, a credit event is defined. One or more of the following can be included as a credit event: bankruptcy or insolvency of the reference entity, failure to pay an amount above a specified threshold over a specified period, and financial or debt restructuring. The protection buyer pays for the protection premium over several settlement dates rather than all at the time of the trade. The payments are typically quarterly and based on the actual/360 convention, the same convention used in the U.S. dollar interest rate swap market.

A single-name CDS has a single reference entity. In the absence of a credit event, the protection buyer will make a quarterly swap premium payment over the life of the swap. If a credit event occurs, two things happen. First, the protection buyer pays out the accrued premium from the last payment date to the time of the credit event, on a days fraction basis. After that payment, there are no further payments of the swap premium by the protection buyer to the protection seller. Second, a termination value is determined for the swap, the procedure depending on the settlement terms specified in the trade's documentation. This will be either physical settlement or cash settlement. As of this writing, the market practice for single-name CDSs is physical settlement. With physical settlement the protection buyer delivers a specified amount of the face value of bonds of the reference entity to the protection seller. The protection seller pays the protection buyer the face value of the bonds.

In a basket default swap, there is more than one reference entity. There are different types of basket default swaps based on when the protection seller is obligated to make a payment to the protection buyer. They are classified as Nth-to-default swaps, subordinate basket default swaps, and senior basket default swap. For example, in an Nth-to-default swap, the protection seller makes a payment to the protection buyer only after there has been a default for the Nth reference entity and no payment for the defaults of the first $(N-1)$ reference entities. Once there is a payout for the Nth reference entity, the swap terminates. Unlike a single-name CDS, the preferred settlement term for a basket default swap is cash settlement. With cash settlement, the protection seller pays the protection buyer an amount equal to the difference between the face value of bonds and their market value after the default.

In a credit default index swap (CDIS), the credit risk of a standardized basket of reference entities is transferred between the protection buyer and protection seller. As of this writing, there are two families of standardized indexes: the Dow Jones CDX and the International Index Company iTraxx. The former includes reference entities in North America and emerging markets, while the latter includes reference entities in Europe and Asia markets. The indexes are updated semiannually as determined by a vote of the index dealers.

The two most actively traded CDIS are the North American Investment Grade Index (CDX NA IG) and the iTraxx Europe Index (iTraxx EUR). The candidate reference entities for the former index are North American companies with an investment-grade rating and the latter European companies with an investment-grade rating. There are 125 corporate names in both indexes and each name is equally weighted within the index (i.e., has a weight of 0.8%). Our focus in this paper will be on the CDX NA IG index.

The index measures the average credit default swap spread of all the index dealers. There are indexes with different maturities. In our empirical analysis, we will focus on the 5-year and 10-year indexes.

The mechanics of a CDIS are slightly different from that of a single-name CDS. As with a single-name CDS, a swap premium is paid. However, if a credit event occurs, the swap premium payment ceases in the case of a single-name CDS. In contrast, for a CDIS the swap payment continues to be made by the protection buyer but based on a reduced notional amount since less reference entities are being protected. As of this writing, the settlement term for a CDIS is physical settlement, although the market is considering moving to cash settlement.

The technology to create a synthetic collateralized debt obligation (CDO) is used to slice a CDIS index into standardized synthetic tranches. As with all structuring technology, the reason for slicing risks is to provide institutional investors with alternative vehicles for obtaining exposure to the risks that are more acceptable to them given their investment objectives and constraints. A CDO is a security backed by a diversified pool of one or more debt obligations such as corporate bonds, corporate loans, emerging market bonds, and structured products (residential and commercial mortgage-backed securities, asset-backed securities, and other CDOs). The manager of the CDO assets is referred to as the collateral manager. The debt obligations issued by the CDO are referred to as tranches or bond classes, with the typical CDO having senior tranches, mezzanine tranches, and subordinate/equity tranche.[1] CDOs are classified as cash CDOs and synthetic CDOs, the latter so named because the collateral manager does not actually own the pool of assets on which it has the credit risk exposure but instead the exposure is obtained by establishing a portfolio of short positions in CDSs.

With the introduction of the CDIS index, a market for standard index tranches has developed. These products, referred to as CDIS index tranches, are not the same as the tranches of a synthetic CDO because they are not constructed by selling a portfolio of CDSs. In addition, index tranches are unfunded and they are a type of CDS, while synthetic CDO tranches are funded (usually in the form of a credit-linked note). However, the net cash flows of a CDIS index tranche and its corresponding synthetic CDO tranche are economically equivalent. Therefore, they are priced in the same way. The most actively traded are those based on the CDX NA IG and the iTraxx EUR. Each tranche is characterized by two percentages. The first percentage is the subordination level. Once the realized losses exceed the subordination level, the investor in that tranche realizes a loss. The second percentage is the upper limit on the credit loss with the loss expressed as a percentage of the size of the underlying

[1] See Lucas, Goodman, and Fabozzi (2006).

reference portfolio. For the CDX NA IG, there are five tranches: an equity tranche 0-3%, a junior mezzanine 3-7%, a 7-10% senior mezzanine, a 10-15% senior, and a 15-30% super senior tranche. For example, for the 7-10% senior mezzanine tranche, an investor in this tranche will only realize a principal loss if there are a sufficient number of defaults for the losses to exceed the subordination of 7% over the life of the tranche and will lose all the principal when the losses reach the upper limit of the tranche of 10%. For the iTraxx-EUR, there are also five tranches and they are 0-3%, 3-6%, 6-9%, 9-12%, and 12-22%.

The following table shows the 5- and 10-year index and tranches market quotes for the Dow Jones CDX NA IG on July 11, 2005 as collected by an investment banking firm:

CDX NA IG (5-year)							
	index	0-3%	3-7%	7-10%	10-15%	15-30%
5-year	58	45.5%	128	128	39	22.5
10-year	82	65.8%	700	157.5	72	36

Unlike a CDO, where the equity tranche is typically held by the CDO sponsor, the equity tranche of a CDX index is also traded in the market. The equity tranche is extremely risky. To reduce the credit risk of the equity tranche, a portion of the equity tranche premium is paid upfront. The market standard of the quarterly premium for the CDX NA IG index equity tranche is 500 basis points per annual.[2] The portion of the equity tranche premium in excess of 500 basis points is paid upfront and the market price change is reflected solely by the upfront payment. Therefore, the equity tranche is quoted by the upfront payment as a percentage of the principal. For example, the market quote of 45.5% for the CDX NA IG 5-year

[2]We asked some major market players why a payment of 500 basis points was selected. No answer was provided.

equity tranche means that the protection buyer pays the protection seller 45.5% of the principal at the time of the trade in addition to the fixed 500 basis points premium per annual on the outstanding principal. For all the non-equity tranches, the market quotes are the premium per annual in basis points, paid quarterly in arrears.

3 One-Factor Copula Model

The critical input in pricing the tranches of a synthetic CDO and a CDIS index is an estimate of the default dependence (default correlation) between the underlying assets. One popular method for estimating the dependence structure is by using copula functions. While there are several types of copula function models, Li (1999, 2000) introduced the one-factor Gaussian copula model for the case of two companies; and Laurent and Gregory (2003) extended the model to the case of N companies. The one-factor Gaussian copula model is now the industry standard model under the assumptions of constant pairwise correlations, constant default rates, and constant recovery rates of 40% conditional upon the occurrence of a default.

Several extensions to the one-factor Gaussian copula model were subsequently proposed. In this section, we provide a general description of the one-factor copula model, followed by a review of three notable models: (1) the random recovery and random factor loadings one-factor Gaussian copula model (Anderson and Sidenius (2004)), (2) the one-factor double t copula model (Hull and White (2004)), and (3) the one-factor normal inverse Gaussian copula model (Kalemanova, Schmid, and Werner (2005)).

Suppose that a CDO includes assets from n companies $i = 1, 2, \ldots, n$ and the default times τ_i of the ith company follows Poisson processes with parameter λ_i.

λ_i is the default intensity of the ith company. Then the probability of a default occurring before time t is

$$P(\tau_i < t) = 1 - exp(-\lambda_i t). \tag{1}$$

In a one-factor copula model, it is assumed that the default time τ_i for the ith company is related to a random variable X_i with a zero mean and a unit variance. For any given time t, there is a corresponding value x such that

$$P(X_i < x) = P(\tau_i < t) \qquad i = 1, 2, \ldots, n. \tag{2}$$

Moreover, the one-factor copula model assumes that each random variable X_i is the sum of two components

$$X_i = a_i M + \sqrt{1 - a_i^2} Z_i, \qquad i = 1, 2, \ldots, n, \tag{3}$$

where Z_i is the idiosyncratic component of company i, and M is the common component of the market. It is assumed that the M and Z_i's are mutually independent random variables. For simplicity, we also assume the random variables M and Z_i's have the same probability distribution. The factor a_i satisfies $-1 \leq a_i \leq 1$. The default time correlation between company i and j is $a_i a_j, (i \neq j)$.

Let F denote the cumulative distribution of the Z_i's and G denote the cumulative distribution of the X_i's. Then given the market variable $M = m$, we have

$$P(Z_i < x | M = m) = F(\frac{x - a_i m}{\sqrt{1 - a_i^2}}) \tag{4}$$

and the conditional default probability is

$$P(\tau_i < t|M = m) = F\{\frac{G^{-1}[P(\tau_i < t)] - a_i m}{\sqrt{1 - a_i^2}}\}. \quad (5)$$

For simplicity, the following two assumptions are made: (1) all companies have the same default intensity, i.e, $\lambda_i = \lambda$ and (2) $a_i = a$ in equation (3). The second assumption means that the contribution of the market component is the same for all the companies and the default time correlation between any two companies is constant, $\beta = a^2$.

Under these homogenous assumptions, given the market situation $M = m$, the conditional default probabilities given in equation (5) are the same for all the companies. For simplicity, we denote the conditional default probability as $D_{t|m}$. Moreover, for a given value of the market component M, the defaults are mutually independent for all the underlying companies. Let $N_{t|m}$ be the total defaults that have occurred by time t conditional on the market condition $M = m$. Then $N_{t|m}$ follows a binomial distribution $Bin(n, D_{t|m})$ and

$$P(N_{t|m} = j) = \frac{n!}{j!(n-j)!} D_{t|m}^j (1 - D_{t|m})^{n-j}, \quad j = 0, 1, 2, \ldots, n. \quad (6)$$

The probability that there will be exactly j defaults by time t is

$$P(N_t = j) = E^M P(N_{t|m}) = \int_{-\infty}^{\infty} P(N_{t|m} = j) f_M(m) dm, \quad (7)$$

where $f_M(m)$ is the probability density function (pdf) of the random variable M.[3]

When the standard normal distribution is used for the distributions of the M and Z_i's in equation (3), the model is the one-factor Gaussian copula model. Andersen

[3]The implementation of the one-factor copula model for the inhomogenous case can be found in Hull and White (2004) and Laurent and Gregory (2003).

and Sidenius (2004) provide two extensions to the one-factor Gaussian copula. In one extension, the random recovery rates are used. Moreover, the recovery rates are inversely dependent on default frequencies to reflect the empirical observation that the recovery rates decrease when the default frequencies increase. In the other extension, the systematic factors a_i are randomized. The simplest case discussed by Andersen and Sidenius is the so-called *two-point distribution*: the a_i's have one value when the market component M is equal to or less than a value, say K, and have a different value when the market component M exceeds K.

Hull and White (2004) extend the one-factor Gaussian copula model to a heavy-tailed copula model by using the Student's t distribution. In their model, the distributions of X_i's are not closed but instead must be calculated. Hull and White find their model fits market prices well when the Student's t distribution has 4 degrees of freedom. However, the result is based only on one day of market data (August 4, 2004).

Kalemanova, Schmid, and Werner (2005) use the heavy-tailed normal-inverse Gaussian (NIG) distribution in the one-factor copula model. The advantage of the NIG distribution is that the X_i's in the model have a closed-form solution. The main purpose of the one-factor NIG copula model is to speed up computation. In their paper, Kalemanova, Schmid, and Werner tested the one-factor NIG copula model for only a single-trading day (April 12, 2006). Compared with the one-factor double t copula model, the one-factor NIG model fits market data a little bit better and it is about five times faster. Whether or not the one-factor NIG model is robust with respect to different time periods needs to be investigated by fitting comprehensive market data to the model, as we do in this paper.

4 Heavy-Tailed Copula Models

The heavy-tailed copula model suggested by Hull and White has two shortcoming. The first is that the tail-fatness cannot be changed continuously. The second is that the maximum tail-fatness occurs when the Student's t distribution has 3 degrees of freedom. In this section, we introduce two new one-factor heavy-tailed copula models: (1) the one-factor double t distribution with fractional degrees of freedom copula model and (2) the one-factor double mixture distribution of t and Gaussian distribution copula model. In each model, there is a parameter to continuously control the tail-fatness of the copula function. Moreover, the maximum tail-fatnesses of our two models are much larger than that for Hull and White's one-factor double t copula model.

4.1 One-Factor Double t Distribution with Fractional Degrees of Freedom Copula Model

Following the common procedure for obtaining the t distribution with integer degrees of freedom, we can obtain the t distribution with fractional degrees of freedom. To explain this, we begin with the gamma(α, β) distribution. The gamma(α, β) has the following pdf

$$f(x|\alpha, \beta) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}} x^{\alpha-1} exp(-x/\beta), \quad 0 < x < \infty, \quad \alpha > 0, \quad \beta > 0. \tag{8}$$

Setting $\alpha = \nu/2$ and $\beta = 2$, we obtain an important special case of the gamma distribution, the chi-square distribution, which has the following pdf:

$$f(x|\nu) = \frac{1}{\Gamma(\nu/2)2^{\nu/2}} x^{\nu/2-1} exp(-x/2), \quad 0 < x < \infty, \quad \nu > 0. \tag{9}$$

If the parameter for degrees of freedom, ν, is an integer, equation (9) is a chi-square distribution with ν degrees of freedom. However, ν need not be an integer. When ν is extended to a positive real number, we obtain the chi-square distribution with ν fractional degrees of freedom.

Assume that (1) U is a standard normal distribution, (2) V is a chi-square distribution with ν fractional degrees of freedom, and (3) U and V are independent. Then based on these assumptions, $T = U/\sqrt{V/\nu}$ has the following pdf

$$f_T(t|\nu) = \frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})\sqrt{\nu\pi}}(1 + t^2/\nu)^{-(\nu+1)/2}, \quad 0 < x < \infty, \quad \nu > 0. \tag{10}$$

This is the Student's t distribution with ν fractional degrees of freedom. The mean and variance of the T are[4]

$$ET = 0, \qquad \nu > 1; \qquad VarT = \frac{\nu}{\nu - 2}, \qquad \nu > 2. \tag{11}$$

For $\nu > 2$, the Student's t distribution in equation (10) can be normalized by making the transition

$$X = \sqrt{(\nu - 2)/\nu}T, \quad \nu > 2. \tag{12}$$

The normalized Student's t distribution with ν ($\nu > 2$) fractional degrees of freedom has the following pdf

$$f_X(x|\nu) = \sqrt{\frac{\nu}{\nu - 2}}\frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})\sqrt{\nu\pi}}(1 + \frac{x^2}{\nu - 2})^{-(\nu+1)/2}, \quad 0 < x < \infty, \quad \nu > 2. \tag{13}$$

Using the normalized Student's t distribution with fractional degrees of freedom for the M and Z_i's in equation (3), we obtain a new extension to the one-factor copula model which we refer to as the *double t distribution with fractional degrees*

[4]See Mardia and Zemroch (1978).

of freedom copula model. In this model, the tail-fatness of the M and Z_i's can be changed continuously by adjusting the fractional degrees of freedom parameter ν. To minimize the number of parameters in the model, the M and Z_i's are restricted so as to have the same fractional degrees of freedom ν, which we refer to as the *tail-fatness parameter of the copula function* in the model.

4.2 One-Factor Double Mixture Distribution of t and Gaussian Distribution Copula Model

In the double t distribution with fractional degrees of freedom copula model, the tail-fatness of the M and Z_i's is controlled by the fractional degrees of freedom parameter of the Student's t distribution. Here, we introduce another distribution for the M and Z_i's, the mixture distribution of Student's t and Gaussian distribution.

Assume U is a normalized Student's t distribution with fractional degrees of freedom and V is a standard normal distribution. We can express a mixture distribution X as

$$X = \begin{cases} U & \text{with probability} \quad 1-p \\ V & \text{with probability} \quad p \end{cases}, \qquad 0 \leq p \leq 1, \tag{14}$$

where p is the proportion of the Gaussian component in the mixture distribution X. The pdf of X is

$$\begin{aligned} f(x) &= \frac{p}{\sqrt{2\pi}} exp(-x^2/2) + \\ &\quad +(1-p)\sqrt{\frac{\nu-2}{\nu}} \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\nu\pi}\Gamma(\nu/2)} (1+\frac{x^2}{\nu-2})^{-(\nu+1)/2}, \end{aligned} \tag{15}$$

where ν is the fractional degrees of freedom of the Student's t distribution.

Using the mixture distribution in equation (15) for the distribution of the M

and Z_i's in equation (3), we obtain one more new extension to the one-factor copula model which we refer to as the *double mixture distribution of t and Gaussian distribution copula model*. In this model, the tail-fatness of the M and Z_i's is controlled by the parameters ν and p. To minimize the number of parameters of the model, the M and Z_i's are confined to have the same ν and p, and ν is fixed. Then the only tail-fatness parameter is p, which we refer to as the *tail-fatness parameter of the copula function* in the model.

5 Model Implementation

Current market practice is to price the tranches of a CDO and a CDIS index in the risk-neutral framework. In this framework, the protection seller and protection buyer of CDIS index tranches are assumed to be indifferent to risk. The basic methodology of risk-neutral pricing is that the expected present value of the swap premium that the protection buyer pays to the protection seller is assumed to be equal to the expected present value of the default compensation payment made by the protection seller to the protection buyer.

In this paper, we implement our heavy-tailed copula models in the risk-neutral framework making the following assumptions:

- All reference entities have the same constant default intensity (implied risk-neutral default intensity).
- The pairwise default time correlations are constant.
- Upon the occurrence of a default, the recovery rate is constant.

In our implementation, the Libor/swap zero curves are used as risk-free interest rates. These zero curves are estimated by the bootstrap methodology using published data for Libor and swap rates.[5] In addition, there are a group of parameters to estimate: recovery rate, default time correlation, implied risk-neutral default intensity, and tail-fatness of the copula function. For the recovery rate and the default time correlation, we use estimates reported in the literature. Based on rating agency studies of corporate defaults, a constant recovery rate of 40% is assumed. The default time correlation is estimated by the best fitting strategy, i.e, the estimated value of the default time correlation is the value that makes our models fit the market data best.

Our approach to estimating the implied risk-neutral default intensity differs from what has been done in other studies. In previous studies, the implied risk-neutral default intensity is considered an exogenous parameter of a one-factor copula model and estimated from market data other than the CDIS index tranches market data. Hull and White (2004), for example, use bond prices or CDS spreads to estimate the implied default intensity. Kalemanova, Schmid, and Werner (2005) use the average portfolio CDS spread to estimate the default intensity. However, the methodology for estimating the implied risk-neutral default intensity is not described in these two papers, and in fact, does not seem to be discussed in the literature as far as we can discern.

In this paper, the implied risk-neutral default intensity is considered an endoge-

[5]There are two interest rates that have been used as a proxy for the risk-free rate: (1) U.S. Treasury zero rates and (2) Libor/swap zero rates. In derivatives pricing, typically the latter are used. The Libor/swap zero rates are calculated from Libor, Eurodollar futures rates, and swap rates. The rates for Libor are available up to 12 months. Typically Eurodollar futures rates are used to produce the Libor/swap zero rates from one year to two years. The Libor/swap zero rates beyond two years are obtained from swap rates. In our case, we use Libor and swap rates to calculate the Libor/swap zero curve. It is accurate enough for our pricing. We have Libor for 3, 6, 9, and 12 months and swap rates for 1, 2, 3, 4, 5, 7, and 10 years. The Libor/swap zero rates at the discrete times of $N \times 0.25$ year ($N = 1, 2, 3, 4$) are derived from Libor. The standard cubic spline method in term structure modeling is used to produce swap rates for the discrete times of $N \times 0.25$ year with (N=5, 6,...,40). Swap rates are rates paid by the fixed-rate payer in exchange for receiving three-month Libor. The Libor/swap zero rates at the discrete times of $N \times 0.25$ year ($N = 5, 6, ..., 40$) are calculated using the bootstrap methodology from the swap rate data.

nous parameter of a one-factor copula model and estimated from the CDIS index tranche quotes directly. The implied risk-neutral default intensity is the one that makes a model fit the market quotes best. This estimating strategy is based on the idea that information about the implied risk-neutral default intensity is embedded in the CDIS index tranche quotes.

The tail-fatness of the copula function parameter is a parameter that we are introducing in this paper and therefore there are no prior studies that we can draw upon for guidance to estimate it. We estimate this parameter in the same way we estimated the implied risk-neutral default intensity and the default time correlation.

In practice, the estimation procedure is to search through the three-dimensional space (of the implied default intensity, the tail-fatness of the copula function, and the default time correlation) to find the combination of the three parameters to satisfy the following criteria:

- Fitting the upfront payment percentage of the equity tranche exactly.
- Minimizing the sum of absolute differences of market spread and fitting spread for all the four non-equity tranches.

In the implementation, we use mesh sizes of 0.001 for the implied risk-neutral default time correlation β and 0.00001 for the implied risk-neutral default intensity λ in our two heavy-tailed copula models. In addition, the mesh size of the tail-fatness parameter is set separately. In the case of a one-factor mixture distribution of t and Gaussian distribution copula model, the mesh size of 0.1 is used for the tail-fatness parameter p. In the case of the one-factor double t distribution with fractional degrees of freedom, the mesh size is dependent on the value of the tail-fatness parameter ν: 0.1 when $\nu \geq 2.5$ and 0.01 if $\nu < 2.5$.

6 Market Data and Empirical Results

The market data for our study, far more comprehensive than in other studies, cover the 5- and 10-year CDX NA IG indexes and tranches from the inception of the market to September 1, 2005. The CDX NA IG indexes are available from October 20, 2003 for both the 5-year and 10-year market data. The 5-year tranches market quotes are available from October 27, 2003 while the 10-year tranches market quotes are available from August 26, 2004. Initially, the index tranches market quotes are not available every trading day; the 5-year index tranches market quotes are available every trading day from June 29, 2004 and the 10-year index tranches market quotes are available every trading day from October 7, 2004.[6]

The CDX NA IG index is updated semiannually.[7] A new version index will be the on-the-run index for the business days from March 20 of a year to September 19 in the same year, or from September 20 of a year to March 19 of the next year. The different version indexes are ordered by serial numbers. The market data we analyze in this paper involves three CDX NA IG index series 3, 4, and 5. The on-the-run periods for these three series are:

- CDX NA IG index Series 3: March 20 2004 $\sim$ September 19, 2004
- CDX NA IG index Series 4: September 20, 2004 $\sim$ March 19, 2005
- CDX NA IG index Series 5: March 20, 2005 $\sim$ September 19, 2005

For each CDX NA IG series, the maturity dates are fixed. For example, the 5-year CDX NA IG index Series 4 has a fixed maturity date of December 20, 2009, and the 10-year CDX NA IG index Series 4 has a fixed maturity date of December

[6] The market quotes were obtained from an investment banking firm that is a major player in the market. As a condition for the use of the data, the authors are not permitted to disclose the source.

[7] http://www.djindexes.com/mdsidx/downloads/credit_derivative/rules.pdf

20, 2014. So the actual time to maturity changes slightly during the on-the-run period. For the 5-year index, the time to maturity ranges from 4.75 years to 5.25 years. For the 10-year index, the time to maturity ranges from 9.75 years to 10.25 years. Our calculation shows that the effect of the slight difference in the time to maturity is quite limited. Consequently, we approximated the time to maturity as 5 years for the 5-year index and 10 years for the 10-year index.

6.1 Overall Fit of Heavy-Tailed Copula Models

We fit our models for the CDX NA IG index (both 5- and 10-year) tranches using daily market data, weekly average market data, and monthly average market data. In general, the overall fit of our models is similar in all the three cases. That is, we find no evidence suggesting that our two models fit one kind of data better than the others. Based on this finding, we use the weekly average market data and the monthly average market data to illustrate the overall fit of our two models.

To check and compare the overall fit of our two models, we computed total absolute pricing errors and the maximum absolute percentage pricing errors. The former is the sum of the absolute differences between the market spreads and model spreads. The latter is the maximum of the absolute pricing errors divided by the market prices. While we do look at both measures, because our pricing strategy is to minimize the total absolute pricing error, that measure is more important than the maximum absolute percentage pricing error.

As we have explained, one major difference of our two models from other one-factor copula models described in the literature is the introduction of the tail-fatness parameter. For a one-factor double t with fractional degrees of freedom copula model, the tail-fatness parameter is the fractional degrees of freedom ν. The tail-fatness increases as the parameter ν decreases. For a one-factor double mixture distribution of t and Gaussian distribution copula model, the tail-fatness parameter is the proportion of Gaussian distribution in the mixture distribution p. The tail-fatness increases as the parameter p decreases.

Tables 1-4 show the fitting results for the monthly average market data from September 2004 to August 2005. Tables 1 and 2 show the fitting results for the 5-year CDX NA IG index tranches. In Table 1, the one-factor double t distribution with fractional degrees of freedom copula model is used. In Table 2, the one-factor double mixture distribution of t and Gaussian distribution copula model is used, where the degrees of freedom of t distribution are fixed as $\nu = 2.1$. It can be seen that the total absolute pricing errors and the maximum percentage pricing errors are quite small. The two models fit the market data well. Comparing the errors of the two models month by month, the total absolute pricing errors in Table 1 are larger than that in Table 2 with one exception (April 2005), and the maximum percentage pricing errors in Table 1 are always larger than that in Table 2. The average difference for the total absolute pricing errors between these two models is 3.0 basis points, while the average difference for the maximum percentage pricing errors between the two models is 4.7%. The one-factor double mixture of t and Gaussian distribution copula model fits the market data slightly better than the one-factor double t distribution with fractional degrees of freedom copula.

The fitting results for the 10-year CDX NA IG index tranches are reported in Tables 3 and 4. Table 3 shows the results for the one-factor double t distribution with fractional degrees of freedom copula model and Table 4 shows the results for the one-factor double mixture distribution of t and Gaussian distribution copula model, where the degrees of freedom of the t distribution are fixed at $\nu = 2.1$. Both models fit the market data well for the first eight months from September 2004 to April 2005. For the last four months from May 2005 to August 2005, the models do not work as well but the results are still good. Based on the total absolute pricing errors, while neither model we propose appears to be superior to the other, for the last four months in our sample period the one-factor double mixture distribution of t and Gaussian distribution copula model fits the market data much better.

The most important result reported in Tables 1-4 is the change of tail-fatness over time. It can be seen that the tail-fatness parameters are time-dependent and change greatly in a one-year period. For the case shown in Table 1, the tail fatness parameter ν is in the range of 2.1∼5.9. Consider that the variance of the t distribution with fractional degrees of freedom is $\frac{\nu}{\nu-2}$. Thus the corresponding variance of the t distribution will have a range of 1.5∼20. This illustrates how large the change of the tail-fatness is in the one-year period. This finding suggests that a one-factor copula model with a fixed tail-fatness parameter may work well for some period of time, but may not work well over time. This problem can be overcome by introducing a tail-fatness parameter into a one-factor copula model as we have done in the models we introduced in this paper.

6.2 Comparison with One-Factor Double t Copula Model

We compared our one-factor double t distribution with fractional degrees of freedom copula model with a one-factor double t copula model introduced by Hull and White (2004) (HW model), where the degrees of freedom of the t distribution are an integer. The HW model is a special case of our model. Because of the advantage of the continuous tail-fatness parameter in our model, one should expect it to perform better than the HW model. Below we report how much of a difference there is. The average weekly market data of the 5-year CDX market data for four weeks are used in the comparison.

A comparison of the two models is shown in Table 5. It can be seen that the total absolute pricing errors of the HW model are at least 5.5 basis points larger than that for our model in the four cases reported in the table. In general, when the tail-fatness parameter in our model falls in the following ranges, $2.0 < \nu \leq 2.9$, $3.2 \leq \nu \leq 3.7$, $4.3 \leq \nu \leq 4.6$ and $5.4 \leq \nu \leq 5.5$, the total absolute pricing error of the HW model is at least double that of our model. When the tail-fatness parameter in our model is out of the ranges specified above, there are two possibilities:

- The tail-fatness parameter in our model is small and close to an integer, for example, in the range of $2.9 < \nu < 3.2$.
- The tail-fatness parameter in our model is greater than 5.5. Because the variance of the t distribution is $\frac{\nu}{\nu-2}$, the difference of the tail-fatnesses for two successive integer degrees of freedom in the HW model is small when ν exceeds 5.5.

In both cases, the difference between our model and the HW model is small. For the extreme situation when the tail-fatness parameter in our model is an integer, the two models give the same results. Otherwise, our model outperforms the HW model.

The maximum tail-fatness of the HW model occurs when the tail-fatness parameter ν is equal to 3. In this case, the variance of the t distribution is 3. By contrast, in our model, the range of tail-fatness is more flexible as the tail-fatness parameter ν can be any real number greater than 2, and the variance of the t distribution with fractional degrees of freedom can go to infinity when the tail-fatness parameter ν approaches 2 from the right side. Our simulation shows that when the tail-fatness parameter ν is less than 2.1, the fitting prices for our model are insensitive to the change of the tail-fatness of the copula function and the computational time increases dramatically. In addition, we find that it is adequate to price market data with the tail-fatness parameter ν greater than or equal to 2.1. So the smallest tail-fatness parameter in our calculation is set as $\nu = 2.1$.

When fitting market data to our model, the tail-fatness parameter can be less than 3. This can be seen in Table 1 where the tail-fatness parameter is below 3 for three months (May, June, and July 2005) in the one-year period. In these three months, the values of the tail-fatness parameter ν are 2.35, 2.13, and 2.1, respectively. Fitting the market data for these three months to the HW model, we find the best fit when ν is equal to 3. However, the total absolute pricing errors for the HW model are much larger than the corresponding total absolute pricing errors for our model. This can be seen in the third case reported in Table 5 where the total absolute pricing error for the HW model is about 23 basis points larger than that for our model and the tail-fatness parameter for our model is $\nu = 2.35$. In general, if the tail-fatness parameter for our model is less than 3, the total absolute pricing error for the HW model increases when the tail-fatness parameter for our model decreases. This means that the total absolute pricing errors for the HW model for May, June, and July 2005 would approximately equal or exceed that reported for the third case in Table 5. One can conclude that for these three months only our model worked.

6.3 Evolution of Tail-Fatness of Copula Function

Market data can be fit to different heavy-tailed copula models. In this paper, the one-factor double t distribution with fractional degrees of freedom copula model and the one-factor double mixture distribution of t and Gaussian distribution copula model have been discussed. The introduction of tail-fatness parameters into these two models make it straightforward to compare the tail-fatnesses inside each model. It is tough to compare the tail-fatnesses between different heavy-tailed copula models because the specific shape of the distribution also makes an important contribution to pricing market data. But, if market data can be accurately priced by using one-factor heavy-tailed copula models with a tail-fatness parameter, then two different heavy-tailed copula models should show the same trend in the evolution of the tail-fatness if both fit market data well.

As can be seen in Tables 1 and 2, both of our heavy-tailed copula models work pretty well for the CDX NA IG 5-year index tranches market data. In Figures 1 and 2, we calculate the evolution of tail-fatness of the copula function for the 5-year CDX NA IG index tranches market data over the one-year period from September 2004 to August 2005. The weekly average market data are used to calculate the tail-fatness of the copula function that makes the models fit market data best.

In Figure 1, where the one-factor double t distribution with fractional degrees of freedom copula model is used for fitting market data, the evolution of the tail-fatness parameter ν is given. The same is done in Figure 2 for the one-factor double mixture distribution of t and Gaussian distribution copula model where the fractional degrees of freedom of the t distribution are fixed at 2.1.

For the market data in the one-year period, a comparison of the two figures shows that the evolutions of the tail-fatness of our two models display the same trends. Initially, the two models have a smaller tail-fatness for about 30 weeks, then there is a W-pattern evolution of the tail-fatness in both figures. The match of the two figures is strong evidence that market data of the CDIS index tranches have a feature that can be captured by heavy-tailed copula models with a tail-fatness parameter.

7 Conclusion

In this paper, we present two new extensions to the one-factor Gaussian copula model for pricing the tranches of a CDO or a CDIS index: a one-factor double t distribution with factional degrees of freedom copula model and a one-factor double mixture distribution of t and Gaussian distribution copula model. For each model, there is a parameter to control the tail-fatness of the copula function. The fitting results of our comprehensive database show that a one-factor copula model with a fixed tail-fatness does not fit market data well over time. However, once we introduce the tail-fatness parameter into our one-factor heavy-tailed copula models, our models do indeed fit market data well over time. The values of the tail-fatness parameter change dramatically over a one-year period. Moreover, for the CDX NA IG 5-year index tranches, the trends in the evolution of tail-fatness of our two models match well, suggesting that there is a feature in market data that can be reflected by heavy-tailed copula models with a tail-fatness parameter.

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Table 1: Fitting results for the one-factor double t distribution with fractional degrees of freedom copula models

Dow Jones CDX NA IG (5-year)						
Tranche	0-3%	3-7%	7-10%	10-15%	15-30%	errors
September 2004	38.4%	261.1	103.9	38.2	11.2	25.8
$\beta = 0.224 \quad \nu = 5.7$	38.4%	262.0	80.7	36.9	11.6	22.3%
October 2004	38.6%	267.8	105.1	38.0	11.7	22.5
$\beta = 0.225 \quad \nu = 5.8$	38.6%	268.0	83.0	37.8	11.7	21.0%
November 2004	34.5%	218.2	85.8	29.7	9.9	21.0
$\beta = 0.224 \quad \nu = 5.6$	34.5%	217.9	66.2	30.7	10.0	22.8%
December 2004	31.2%	181.8	61.4	23.7	8.1	10.2
$\beta = 0.222 \quad \nu = 5.6$	31.2%	181.2	54.4	25.7	8.7	11.4%
January 2005	32.2%	196.8	67.7	24.7	8.8	11.0
$\beta = 0.222 \quad \nu = 5.7$	32.2%	196.6	59.0	27.2	8.8	10.1%
February 2005	30.3%	176.5	59.7	20.5	7.6	11.7
$\beta = 0.217 \quad \nu = 5.8$	30.3%	177.2	51.9	23.7	7.6	15.6%
March 2005	29.6%	168.5	55.1	20.1	7.9	9.7
$\beta = 0.219 \quad \nu = 5.4$	29.6%	167.7	49.7	23.5	8.0	16.9%
April 2005	38.1%	214.8	60.2	27	10.8	3.3
$\beta = 0.196 \quad \nu = 4.7$	38.1%	215.1	59.1	27.6	9.5	12.0%
May 2005	52.5%	226.1	55.4	26.4	13.9	10.4
$\beta = 0.129 \quad \nu = 2.35$	52.5%	226.3	45.7	26.1	14.1	17.5%
June 2005	46.6%	153.4	44.3	23.0	13.7	10.4
$\beta = 0.131 \quad \nu = 2.13$	46.6%	154.1	35.2	22.6	13.9	20.5%
July 2005	43.8%	123.7	38.5	21.0	14.0	16.2
$\beta = 0.133 \quad \nu = 2.1$	43.8%	132.8	32.3	21.4	13.5	16.1%
August 2005	38.9%	134.3	37.8	19.8	11.0	5.0
$\beta = 0.155 \quad \nu = 2.8$	38.9%	134.9	34.1	20.0	10.4	9.8%

Table 2: Fitting results for one-factor double mixture distribution of t and Gaussian distribution copula models

Dow Jones CDX NA IG (5-year)						
Tranche	0-3%	3-7%	7-10%	10-15%	15-30%	errors
September 2004	38.4%	261.1	103.9	38.2	11.2	21.8
$\beta = 0.269 \quad P = 0.52$	38.4%	260.3	85.0	36.3	11.4	18.2%
October 2004	38.6%	267.8	105.1	38.0	11.7	18.1
$\beta = 0.270 \quad P = 0.53$	38.6%	267.2	88.1	37.6	11.6	16.2%
November 2004	34.5%	218.2	85.8	29.7	9.9	17.1
$\beta = 0.268 \quad P = 0.50$	34.5%	218.3	69.7	29.5	9.7	18.7%
December 2004	31.2%	181.8	61.4	23.7	8.1	6.2
$\beta = 0.266 \quad P = 0.47$	31.2%	181.4	56.1	23.9	8.4	8.6%
January 2005	32.2%	196.8	67.7	24.7	8.8	8.1
$\beta = 0.268 \quad P = 0.49$	32.2%	197.7	62.2	26.4	8.8	8.1%
February 2005	30.3%	176.5	59.7	20.5	7.6	8.3
$\beta = 0.262 \quad P = 0.48$	30.3%	177.0	53.8	22.4	7.6	9.9%
March 2005	29.6%	168.5	55.1	20.1	7.9	6.2
$\beta = 0.263 \quad P = 0.46$	29.6%	168.2	51.0	21.7	7.7	7.7%
April 2005	38.1%	214.8	60.2	27	10.8	4.2
$\beta = 0.251 \quad P = 0.43$	38.1%	214.7	63.1	27.6	10.2	5.6%
May 2005	52.5%	226.1	55.4	26.4	13.9	8.3
$\beta = 0.176 \quad P = 0.19$	52.5%	226.9	48.7	27.0	13.7	12.1%
June 2005	46.6%	153.4	44.3	23.0	13.7	8.0
$\beta = 0.164 \quad P = 0.10$	46.6%	153.8	37.7	23.9	13.8	14.9%
July 2005	43.8%	123.7	38.5	21.0	14.0	12.5
$\beta = 0.141 \quad P = 0.04$	43.8%	129.0	32.7	21.7	13.3	15.1%
August 2005	38.9%	134.5	37.8	19.8	11.0	2.6
$\beta = 0.211 \quad P = 0.21$	38.9%	134.5	36.3	20.8	11.1	5.1%

Table 3: Fitting results for one-factor double t distribution with fractional degrees of freedom copula models

Dow Jones CDX NA IG (10-year)						
Tranche	0-3%	3-7%	7-10%	10-15%	15-30%	errors
September 2004	56.4%	600.7	279.4	139.5	43.3	6.6
$\beta = 0.256 \quad \nu = 8.2$	56.4%	600.9	277.8	141.0	40.3	7.9%
October 2004	56.9%	576.1	264.2	129.3	44.9	13.3
$\beta = 0.256 \quad \nu = 6.2$	56.9%	576.4	256.9	129.3	39.2	10.5%
November 2004	54.6%	526.9	210.8	97.9	34.9	24.5
$\beta = 0.228 \quad \nu = 7.6$	54.6%	526.7	221.2	104.0	27.1	22.4%
December 2004	54.1%	479.1	157.6	78.3	28.6	28.4
$\beta = 0.208 \quad \nu = 6.6$	54.1%	478.6	184.1	81.7	20.6	28.0%
January 2005	57.5%	516.9	195.2	90.8	34.6	14.4
$\beta = 0.226 \quad \nu = 4.4$	57.5%	515.6	200.0	94.1	29.6	14.5%
February 2005	57.1%	493.8	186.5	87.2	33.9	6.4
$\beta = 0.226 \quad \nu = 4.0$	57.1%	494.2	186.0	87.7	28.9	14.8%
March 2005	57.1%	471.6	168.8	77.8	32.5	12.5
$\beta = 0.220 \quad \nu = 3.4$	57.1%	472.0	167.0	78.7	28.1	13.5%
April 2005	61.4%	563.1	213.2	93.5	38.4	16.1
$\beta = 0.210 \quad \nu = 3.5$	61.4%	563.9	207.3	95.2	31.7	17.5%
May 2005	69.1%	807.3	202.8	78.4	39.2	155.3
$\beta = 0.166 \quad \nu = 2.14$	69.1%	663.9	202.8	85.2	34.0	17.8%
June 2005	65.3%	692.4	175.0	63.2	32.7	138.9
$\beta = 0.172 \quad \nu = 2.60$	65.3%	570.4	175.5	75.4	27.7	18.7%
July 2005	64.3%	653.6	144.7	72.3	36.0	144.1
$\beta = 0.172 \quad \nu = 2.60$	64.3%	552.2	167.9	72.0	26.8	25.6%
August 2005	62.4%	594.0	137.8	69.8	30.7	97.3
$\beta = 0.165 \quad \nu = 3.3$	62.4%	537.0	168.9	69.6	21.7	29.3%

Table 4: Fitting results for one-factor double mixture distribution of t and Gaussian distribution copula models

Dow Jones CDX NA IG (10-year)						
Tranche	0-3%	3-7%	7-10%	10-15%	15-30%	errors
September 2004	56.4%	600.7	279.4	139.5	43.3	39.7
$\beta = 0.301$ $P = 0.69$	56.4%	607.5	255.8	132.1	45.3	8.5%
October 2004	56.9%	576.1	264.2	129.3	44.9	50.0
$\beta = 0.298$ $P = 0.68$	56.9%	608.8	252.5	129.3	44.3	4.4%
November 2004	54.6%	526.9	210.8	97.9	34.9	12.1
$\beta = 0.285$ $P = 0.64$	54.6%	525.8	202.8	100.5	33.6	3.8%
December 2004	54.1%	479.1	157.6	78.3	28.6	18.8
$\beta = 0.273$ $P = 0.59$	54.1%	478.1	170.7	82.3	27.9	8.3%
January 2005	57.5%	516.9	195.2	90.8	34.6	25.4
$\beta = 0.279$ $P = 0.52$	57.5%	517.7	175.5	86.4	34.1	10.1%
February 2005	57.1%	493.8	186.5	87.2	33.9	31.0
$\beta = 0.282$ $P = 0.49$	57.1%	497.1	164.1	82.0	34.0	12.0%
March 2005	57.1%	471.6	168.8	77.8	32.5	28.7
$\beta = 0.275$ $P = 0.44$	57.1%	474.7	147.8	74.0	32.6	12.9%
April 2005	61.4%	563.1	213.2	93.5	38.4	47.9
$\beta = 0.274$ $P = 0.43$	61.4%	568.1	177.5	88.0	40.1	16.7%
May 2005	69.1%	807.3	202.8	78.4	39.2	96.7
$\beta = 0.195$ $P = 0.48$	69.1%	761.4	231.6	92.1	30.9	20.9%
June 2005	65.3%	692.4	175.4	63.2	32.7	85.7
$\beta = 0.200$ $P = 0.54$	65.3%	663.4	204.1	82.9	24.4	31.2%
July 2005	64.3%	653.6	144.7	72.3	36.0	83.4
$\beta = 0.200$ $P = 0.49$	64.3%	614.5	176.2	72.2	23.3	35.3%
August 2005	62.4%	594.0	137.8	69.8	30.7	51.5
$\beta = 0.199$ $P = 0.53$	62.4%	586.4	171.6	69.4	20.1	34.5%

Table 5: Comparison of the overall fit of the one-factor double t distribution with fractional degrees of freedom copula model and the one-factor double t copula model

Dow Jones CDX NA IG (5-year)						
March 28, 2005 ∼ April 1, 2005						
	0-3%	3-7%	7-10%	10-15%	15-30%	errors
market Data	31.3%	186.4	60.5	24.0	9.0	
$\beta = 0.221 \quad \nu = 5.5$	31.3%	186.9	55.9	26.1	8.7	7.5
$\beta = 0.219 \quad \nu = 6.0$	31.3%	193.5	57.2	25.9	8.0	13.3
April 25, 2005 ∼ April 29, 2005						
	0-3%	3-7%	7-10%	10-15%	15-30%	errors
Market Data	41.2%	231.5	61.9	29.8	11.9	
$\beta = 0.190 \quad \nu = 4.3$	41.2%	231.4	62.2	29.3	10.9	2.1
$\beta = 0.193 \quad \nu = 4$	41.2%	224.7	61.3	30.0	11.7	11.7
August 1, 2005 ∼ August 5, 2005						
	0-3%	3-7%	7-10%	10-15%	15-30%	errors
Market Data	39.8%	113.0	34.1	17.9	11.8	
$\beta = 0.133 \quad \nu = 2.35$	39.8%	113.3	28.0	17.8	10.6	7.7
$\beta = 0.137 \quad \nu = 3$	39.8%	135.9	30.6	16.8	8.2	31.1
August 22, 2005 ∼ August 26, 2005						
	0-3%	3-7%	7-10%	10-15%	15-30%	errors
Market Data	38.0%	123.0	35.3	17.9	9.0	
$\beta = 0.147 \quad \nu = 2.8$	38.0%	123.0	30.4	17.8	9.3	5.3
$\beta = 0.148 \quad \nu = 3$	39.1%	129.5	31.3	17.5	8.7	11.2

Figure 1: Evolution of the tail-fatness for the one-factor double t distribution with fractional degrees of freedom copula model

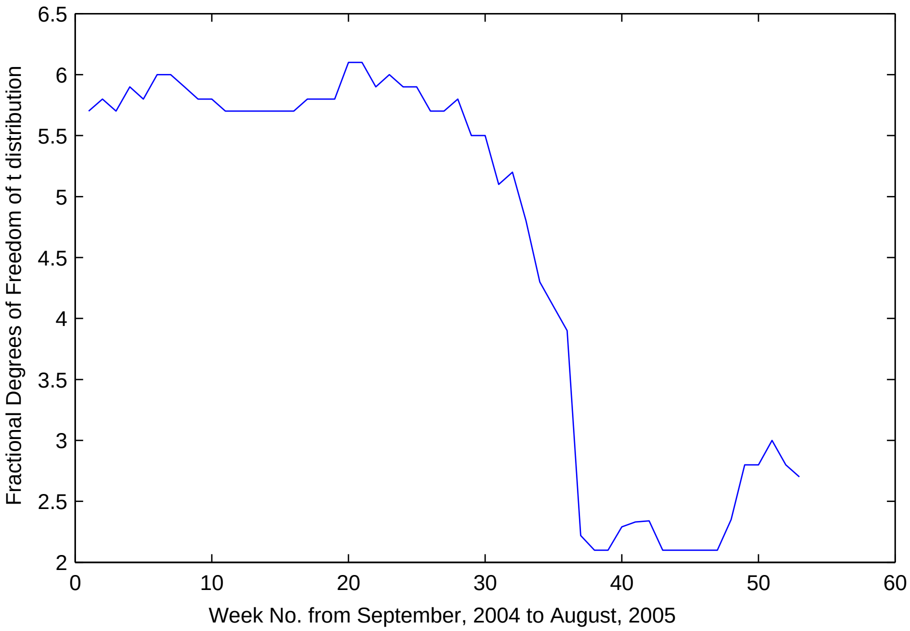

Figure 2: Evolution of the tail-fatness for the one-factor double mixture distribution of t and Gaussian distribution copula model