Stochastic technical analysis for decision making on the financial market

(October 21, 2014)

Markus Höchstötter[1,+], Mher Safarian[1]

Abstract

We apply the well-known CUSUM and the Girshick-Rubin algorithm as trading strategies involving only mutually exclusive long positions in cash and the DAX at Frankfurt mid-day auction prices. We select optimal pairs of fixed thresholds for up- and down- movements from a pre-defined two-dimensional grid, hence, admitting asymmetric intervals. We show that under three different scenarios for transaction costs, the CUSUM technique not only outperforms the passive investment in the DAX but also the alternative Girshick-Rubin algorithm.

Keywords: CUSUM, Girshick-Rubin, trading algorithm, DAX

1. Introduction

One of the most critical questions in asset management and investing is the detection of changes in the current regime. The theoretical terminology refers to this as change-point detection or break-point analysis. In an economic context, mod-

[1] Econometrics and Statistics. School of Industrial Engineering. KIT. Schlossbezirk 12. D-76128 Karlsruhe.

[+] Corresponding author: `markus.hoechstoetter@partner.kit.edu` and `markus@hoechstoetter.com`

We thank Torsten Lüdecke and Wolf-Dieter Heller for their valuable support as well as several attendees at the IFORS 2014, Barcelona, for their valuable comments.

els often involve a multitude of parameters, the stability of which over time has been put into question at least since Isaac and Griffin (1989). Many others such as, for example, Balding et al. (2008), Hamilton and Susmel (1994), Schaller and Van Norden (1997), Bai and Perron (1998), Hansen (2000), Dias and Embrechts (2002), Western and Kleykamp (2004) followed suit. These approaches detect change points by looking into the rearview mirror, that is, they analyze historical time series and determine the most probable scenario concerning a change in value, or multiple changes of a particular parameter of a more or less complex model, in the past. This, however, is of limited to no value to an investor or trader who has to receive signals immediately if a change appears likely.

Interestingly, a suitable approach has been provided by some technique developed for quality control in manufacturing, i.e. control chart techniques first developed such as Shewhart (1932). The general idea is to observe some time series until a predefined threshold is trespassed. Page (1954a) and Page (1954b) coined the term CUSUM as short for cumulative-sum where the actual value of a process, for example, a random walk is compared to some prior extreme value such as an all-time low or high, respectively. If the difference between actual and reference value is greater than the threshold, a signal is delivered. The initial approach was augmented by the moving average control chart by Roberts (2000).

The CUSUM is equivalent to the filter trading rule introduced by Alexander (1961). Initial results of this rule are given, for example, by Alexander (1961), Fama and Blume (1966), or Dryden (1969) who showed that, after consideration of trading costs, the filter method cannot outperform the traditional buy-and-hold strategy. Moreover, a shortcoming of the original rule was detected in that, under certain circumstances, the trading rule could result in unbounded losses.

In financial applications, methods of quickest detection of a change-point are of interest that are "free" of a distribution of a random sequence, i.e. nonparametric methods.[1] The biggest problem with control techniques is the proper determination of the threshold which my actually vary over time. Solutions in that context are provided by, for example, Verdier et al. (2008). It was shown that when the change-point is a random variable with known distribution, then the optimal method is to observe an à posteriori probability of a change-point until it reaches some threshold value which may be analytically calculated. This method, however, cannot be applied to problems arising in practice since it is almost always impossible to obtain any à priori information on the distribution of the time of occurrence of a change-point itself as well as on the distribution of a random sequence before and after the change. This, for example, makes the approach by Luo et al. (2009) who suggest variable sampling intervals under known distributions inapplicable, in our context. The most flexible approach so far has been suggested by Jeske et al. (2009) with, however, the still slightly unrealistic assumption of independent observations. A good overview of the topic is given by Wald (2013) and Shiryaev (2007).

2. Algorithms

Hereafter, the scheme of observations we deal with is as follows. Let $(\Omega, \mathcal{F}, P)$ be a probability space on which there is defined a random sequence $x = (x_i)_{i\geq 0}$ with

$$x_i = a + \xi_i \, I(i < i^*) + (c + \eta_i) \, I(i \geq i^*)$$

[1] It turns out that it is possible to give "nonparametric" versions to some popular parametric methods.

where $\xi = (\xi_i)_{i\geq 0}$ and $\eta = (\eta_i)_{i\geq 0}$ are random sequences such that $E(\xi_i) = E(\eta_i) = 0$ and a and c are constants with $a(a+c) < 0$. The index value i^* indicates a change-point.

The objective is to minimize the average delay until the detection of a true change-point while, at the same time, keeping the number of false alarms down. Among the algorithms we will present, there is no universal one for the quickest detection of a change-point in a variety of settings. Each one outperforms in its "domain".

Let

- initial value of time interval: T_0
- final value of time interval: T_1
- tick times of the asset price: $(\tau_j)_{j=0}^N$
- best bid price of the asset at tick: $(S_{\tau_j}^{bid})_{j=0}^N$
- best ask price of the asset at tick: $(S_{\tau_j}^{ask})_{j=0}^N$
- parameter of partition of time interval: Λ
- interval for smoothing: M
- coefficient of smoothing: $\alpha,\ 0 < \alpha < 1$
- threshold value of the algorithm: h

Let $L = [(T_1 - T_0)/\Lambda]$.[2] We define values of best bid and best ask asset prices at the points of (equidistant) partition $t_k = k\Lambda, (0 \leq k \leq L)$ of the time interval

[2] Here, $[a]$ stands for an integer part of a.

$[T_0, T_1]$ as values $S^{bid}_{\tau_m}$ and $S^{ask}_{\tau_m}$, respectively, at the tick times:

$$\tau_m = \max_{0 \le j \le N} \{\tau_j : \tau_j \le t_k\}.$$

In the following, by $S_k := S^{bid}_k - S^{ask}_k$, we will denote the mid-day auction prices with zero spread and the assumption of unlimited liquidity, instead. Also, we define

$$\xi_j = \ln S_j - \ln S_{j-1}, 1 \le j \le L.$$

2.1. CUSUM

Let k_0 be the last point in time that the signal (of a true change-point) was detected($k_0 = T_0$, at the beginning). Then, define recurrently

$$\begin{aligned} R^{(1)}_{k_0} &= R^{(2)}_{k_0} = 0 \\ R^{(1)}_k &= (R^{(1)}_{k-1} + \xi_k)^+ \\ R^{(2)}_k &= (-R^{(2)}_{k-1} + \xi_k)^-, \; k_0 < k \le L \end{aligned}$$

where we have used the convention $(a)^+ = \max\{a, 0\}$ and $(a)^- = -\min\{a, 0\}$. If $R^{(1)} \ge h$, for the first time since the last signal and for some pre-defined threshold $h > 0$, then, the algorithm indicates that the random sequence under observation shows an up-trend and, thus, sends a "buy" signal. If, on the other hand, $R^{(2)} \ge h$, then the random sequence shows a down-trend and the algorithm sends a "sell" signal.

2.2. Girshick-Rubin

Let k_0 be the last point in time that a signal was detected. Define

$$\begin{aligned} R^{(1)}_{k_0} &= R^{(2)}_{k_0} = 0 \\ R^{(1)}_k &= 1/(k+1)\exp(\xi_k)(1 + kR^{(1)}_{k-1}) \\ R^{(2)}_k &= 1/(k+1)\exp(-\xi_k)(1 + kR^{(2)}_{k-1}), \; k_0 < k \le L \end{aligned}$$

If $R^{(1)} \geq h$, for some pre-defined threshold $h > 0$, then, there is a signal to buy the asset. On the other hand, if $R^{(2)} \geq h$, then there is a signal to sell the asset.

2.3. Concluding remarks

At this point, it ought to be mentioned that the threshold value of the algorithm h as far as the other parameters Λ, α, and c should be determined by trial-and-error methods or grid search. For simplicity, we only assume h to be variable and keep the other parameters fixed ($\alpha = -1$, $c = +2$, and $\Lambda = 1$).[3] To provide for greater flexibility, we allow h for up-movements to be different from h for down-movements.

3. Data

We obtained the daily Frankfurt mid-day auction prices (*Kassakurse* of the DAX index (WKN 846900) from the Karlsruher Kapitalmarktdatenbank. Our sample covers the period between January 1, 1990 and March 31, 2014, yielding over 6100 observations. Thus, it contains observations from a selection of different crises such as, for example, the Japanese crisis in 1997, the Rubel crisis in 1998, the dot-com bubble in 2000 and, of course, the latest crisis beginning 2007. Figure 1 displays a chart of the DAX level in that period. The currency is euros implying a backward conversion of the prices in deutsche mark prior to January 1, 2002. Starting at a level of EUR 1814, the DAX gained EUR 7741 over the sample period resulting in a level of EUR 9556 at the end of March, 2014. This is roughly 5.3 times the level at the beginning of the sample period.

[3] As of yet, we do not (!) have a preferred algorithm for the determination of these parameters.

—

Figure 1 here

—

In Figure 2, we display the daily log-returns of the DAX over the sample period. The minimum is -0.0987 and the maximum 0.1080 with a mean of nearly zero. The standard deviation is 0.0146 while skewness and kurtosis are -0.1244 and 7.8817, respectively. This hints at a strongly leptocurtic and asymmetric distribution of the log-returns which is supported by the very high Jarque-Bera test statistic of 6098. The 0.05- and 0.95-quantiles are equal to -0.0233 and 0.02183, respectively. A kernel density plot is given by Figure 3.

—

Figure 2 here

—

—

Figure 3 here

—

4. Set-up and Results

Our approach is as follows. We start with an initial cash position (long) of 1'000 euros at the beginning, on January 1, 1990. At the end, on March 31, 2014, we dissolve any investment at current value if a long position is taken; otherwise, we simply consider the cash position. In between, we either invest the entire current amount at the current price of the DAX (if a buy-signal occurs) and hold it until the next sell-signal or sell everything and hold cash only (if a sell-signal occurs). Thus, portfolio weights for cash and asset are mutually exclusively either zero or one. The positions are held, respectively, until the first signal of opposite sign is observed. Hence, consecutive signals of same sign do not lead to any action.

Further, we assume three scenarios. In the first scenario, there are no transaction costs. So, we alter positions at zero expense. Under the second scenario, transactions are assumed to be EUR 10 for both sell and buy. The third scenario has transaction fees of EUR 30 for both sell and buy. Under the latter two scenarios, we assume fixed transaction costs independent of size.[4]

Since we are unaware of an analytical optimal solution for threshold parameter h when dealing with these DAX log-returns, we resort to grids for both algorithms, i.e., CUSUM in subsection 2.1 as well as Girshick-Rubin in subsec-

[4] We admit that this may be somewhat unrealistic especially for the earlier years of the sample period. On average, the EUR 10 fee of the second scenario most likely represents reality most adequately given the trading fees universe of German traders within the last few years. Also, we assume unlimited liquidity on both the sell and buy side to hold prices stable. We concede that this might leave room for improvement. However, we do not think that this will impair the overall picture obtained from our analysis.

tion 2.2. To be precise, we use two different grids. The coarser one covers the domain $h \in [0.05, 0.50]$ with step size 0.05. The finer one covers the domain $h \in [0.01, 0.10]$ with step size 0.01. To provide greater flexibility, as mentioned before, we allow the bounds to be asymmetrical, that is, the threshold h for up-movements (which we denote h_1, here) can be different from the threshold for down-movements (which we denote h_2, here).[5] As computer software, we used Matlab. —

Figure 4 here

—

First, we discuss the results for the coarser grid between 0.05 and 0.50 with step size 0.05. In Figure 4, sorted by scenario, we display the surface of the end of the period values of the CUSUM strategy for the different threshold values h_1 and h_2. By looking at scenario one in Figure 4a, it becomes obvious that the final pay-off increases for smaller thresholds. As can be see from Table 1, the maximum over this grid is roughly 2.1 million euros which is attained for the symmetric case $h_1 = h_2 = 0.05$. While still in the corner of the lowest threshold values, the introduction of fees in scenarios two and three definitely results in a reduction of the final pay-off, as can be seen in Figure 4a and Figure 4a, respectively. The optimal pay-off under scenario one is ca. 1,2 million euros and under scenario three, it equals 27,899.19 euros. From Table 1, we can see that for the highest fee, optimal choice is asymmetrical with $h_1 = 0.05$ and $h_2 = 0.10$. —

[5] One should keep in mind that symmetrical bounds translate into asymmetrical bounds for actual DAX level changes since we focus on log-returns.

Figure 5 here

—

For the same grid, the final pay-offs of the Girshick-Rubin strategies are displayed in Figure 5. While pay-offs are generally lower than those from CUSUM, it also seems as if there is no tendency for symmetry and lower values of the thresholds h_1 and h_2 as was the case before. For the highest fees, as can be seen in Figure 5c, the strategies with large threshold values clearly dominate the lower values. In detail, the respective optimal final pay-offs in each scenario are 7,331.32 euros and 2,951.79 euros, respectively, as also reported in Table 1.

—

Figure 6 here

—

From Figure 4, we realize that we have to analyze the region for very small thresholds, for the CUSUM strategy. For this reason, we compute final pay-offs over the fine grid between 0.01 and 0.10 with step size 0.01. The resulting pay-off surfaces sorted by scenario are presented in Figure 6. In Figure 6a, we see that under the first scenario, the optimal strategy is for the smallest values of the threshold in that grid which supports our intuition from the previous plots. In fact, from Table 1, the optimal strategy yields a pay-off of over 133 trillion (!) euros which is definitely owed to the unrealistic setting of no transactions costs. The optimal threshold values are symmetrically located at $h_1 = h_2 = 0.01$. Under scenario two, as shown in Figure 6b, the picture is similar, however with a smaller final pay-off of 54 trillion (!) euros due to 10 euros transaction fees. And finally, in Figure 6c, we see the results for the third scenario. Here, due to the higher transaction fees of 30 euros, the optimal final pay-off of 80,957.02 euros has been strongly deflated compared to the previous two settings. The optimal threshold is asymmetric with $h_1 = 0.03$ and $h_2 = 0.09$, respectively, and clearly leaning away from the lowest corner.

—

Figure 7 here

—

The results of the Girshick-Rubin algorithm are presented in Figure 7. In Figure 7a, we see that the final pay-off under scenario one peaks near the lowest threshold values. To be precise, the optimal pay-off of 30,580.40 euros is achieved for $h_1 = 0.01$ and $h_2 = 0.02$, respectively. With transaction fees of 10 euros, the prospects are generally dim. In the best case, for high threshold values near 0.1 there are final pay-offs of roughly 500 euros which represents a loss given that we started with 1,000 euros. In any other area of the grid, performing the algorithm leads to ruin. So, here, it is generally better to keep the initial cash position of 1,000 euros which is the strategy listed in Table 1. For transactions fees of 30 euros, things are even worse. Any combinations of threshold values leads to ruin. So again, we should rather keep the 1,000 euros for the entire period.

From Figure 5b, we might come to the conclusion that an optimal set of thresholds for the Girshick-Rubin algorithm can be found somewhere between 0.20 and

0.40. To validate this hypothesis, we concentrate another grid on this range with step size 0.01. The optimal final pay-offs under each scenario are listed at the bottom in Table 1. When transaction fees are equal to 20 euros, then the best strategy, so far, has been detected to lie inside of this grid at $h_1 = 0.20, h_2 = 0.39$ which is well inside the coarse grid between 0.05 and 0.50 and only slightly away from a grid point there. This gives rise to the notion that some optimum might be located in this areas that could be verified by an even finer grid.

5. Summary

We have presented two different trading algorithms based on the well-known CUSUM technique as well as the Girshick-Rubin algorithm, respectively. We selected the optimal pairs of possibly asymmetric thresholds from a two-dimensional grid. While we admit that we make some naïve assumptions concerning unlimited liquidity at the Frankfurt mid-day auction, we provide for some realistic constraints by introducing several constraints in the form of scenarios for trading fees. We witness the problem of unboundedness in case of the CUSUM algorithm.

Overall, we can say that the CUSUM algorithm yields superior results. Generally, its optimal thresholds are symmetric and located in the lower region of the grid. The alternative Girshick-Rubin strategy still outperformed the passive long investment in the DAX. However, it is always dominated by the CUSUM strategy. Moreover, in a few scenarios, the Girshick-Rubin algorithm led to ruin.

Alexander, S. (1961). Price Movements in Seculative Markets: Trends or Random Walk. *Industrial Management Review 2*, 7–26.

Bai, J. and P. Perron (1998). Estimating and Testing Linear Models with Multiple Structural Changes. *Econometrica 66*(1), 47–78.

Balding, D. J., N. A. C. Cressie, G. M. Fitzmaurice, I. M. Johnstone, G. Molenbergh, D. W. Scott, A. F.M.Smith, R. A. Tsay, and S. Weisberg (Eds.) (2008). *Time Series Analysis: Forecasting and Control* (4 ed.). WILEY SERIES IN PROBABILITY AND STATISTICS. 111 River Street, Hoboken, New Jersey, USA.: Wiley.

Dias, A. and P. Embrechts (2002). Chang-Poiint Analysis for Dependence Structures in Finance and Insurance. Technical report, ETH Zurich.

Dryden, M. M. (1969). Share Price Movements: A Markovian Approach. *The Journal of Finance 24*(1), 49–60.

Fama, E. F. and M. E. Blume (1966, January). Filter Rules and Stock-Market Trading. *The Journal of Business 39*(1), 226–241.

Hamilton, J. D. and R. Susmel (1994). Autoregressive Conditional Heteroskedasticity and Changes in Regime. *Journal of Econometrics 64*, 307–333.

Hansen, B. E. (2000). Testing for Structural Change in Conditional Models. *Journal of Econometrics 97*(1), 93–115.

Isaac, L. W. and L. J. Griffin (1989). A historicism in time-series analyses of historical process: Critique, redirection, and illustrations from U.S. labor history. *American Sociological Review 54*(6), 873–890.

Jeske, D. R., V. Montes De Oca, W. Bischoff, and M. Marvasti (2009). Cusum techniques for timeslot sequences with applications to network suveillance. *Computational Statistics and Data Analysis 53*, 4332–4344.

Luo, Y., Z. Li, and Z. Wang (2009). Adaptive CUSUM control chart with variable sampling intervals. *Computational Statistics and Data Analysis 53*, 2693–2701.

Page, E. (1954a). Continuous Inspection Schemes. *Biometrika 41*(1/2), 100–115.

Page, E. (1954b). An Improvement to Wald's Approximation for Some Properties of Sequential Tests. *Journal of the Royal Statistical Society. Series B 16*(1), 136–139.

Roberts, S. W. (2000). Control Chart Tests Based on Geometric Moving Averages. *Technometrics 42*(1), 97–101.

Schaller, H. and S. Van Norden (1997). Regime Switching in Stock Market Returns. *Applied Financial Economics 7*(2), 177–191.

Shewhart, W. A. (1932). The Economic Control of Quality of Manufactured Product. *Journal of the Royal Statistical Society 95*(3), 546–549.

Shiryaev, A. N. (2007). *Optimal Stopping Rules*. Berlin: Springer.

Verdier, G., N. Hilgert, and J.-P. Vila (2008). Adaptive threshold computation for CUSUM-type procedures in change detection and isolation problems. *Computational Statistics and Data Analysis 52*, 4161–4174.

Wald, A. (2013). *Sequential Analysis*. Dover Publications, Inc.

Western, B. and M. Kleykamp (2004). A Bayesian Change Point Model for Historical Time Series Analysis. Technical report, Princeton.

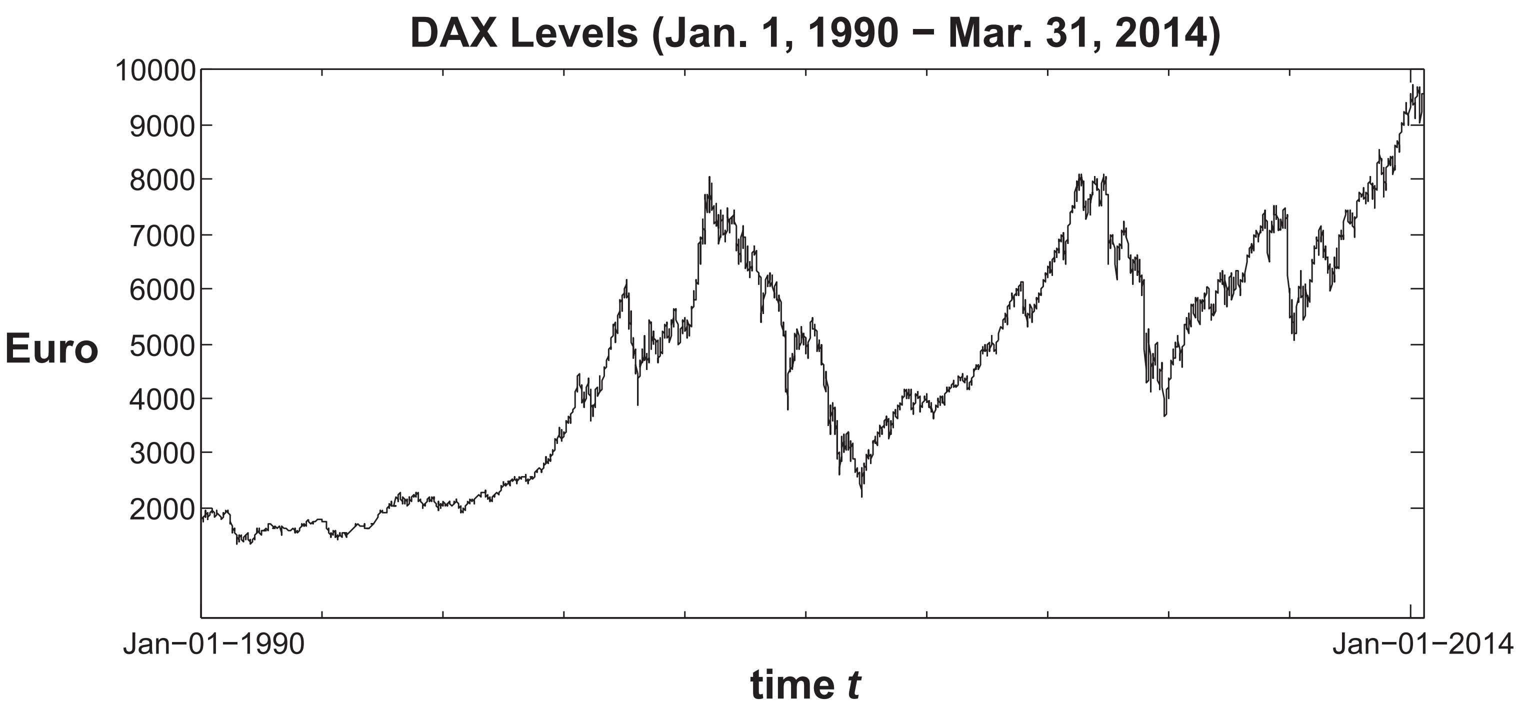


Figure 1: DAX levels.

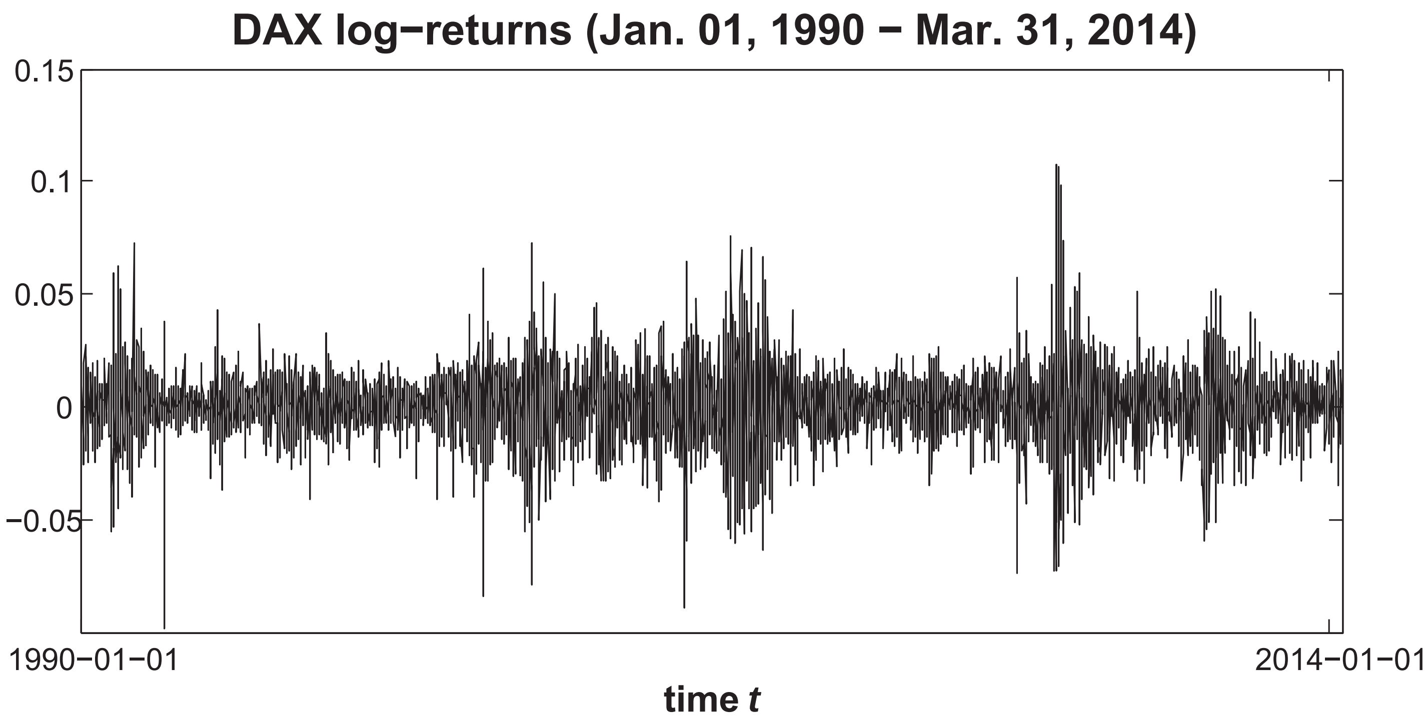


Figure 2: DAX log-returns.

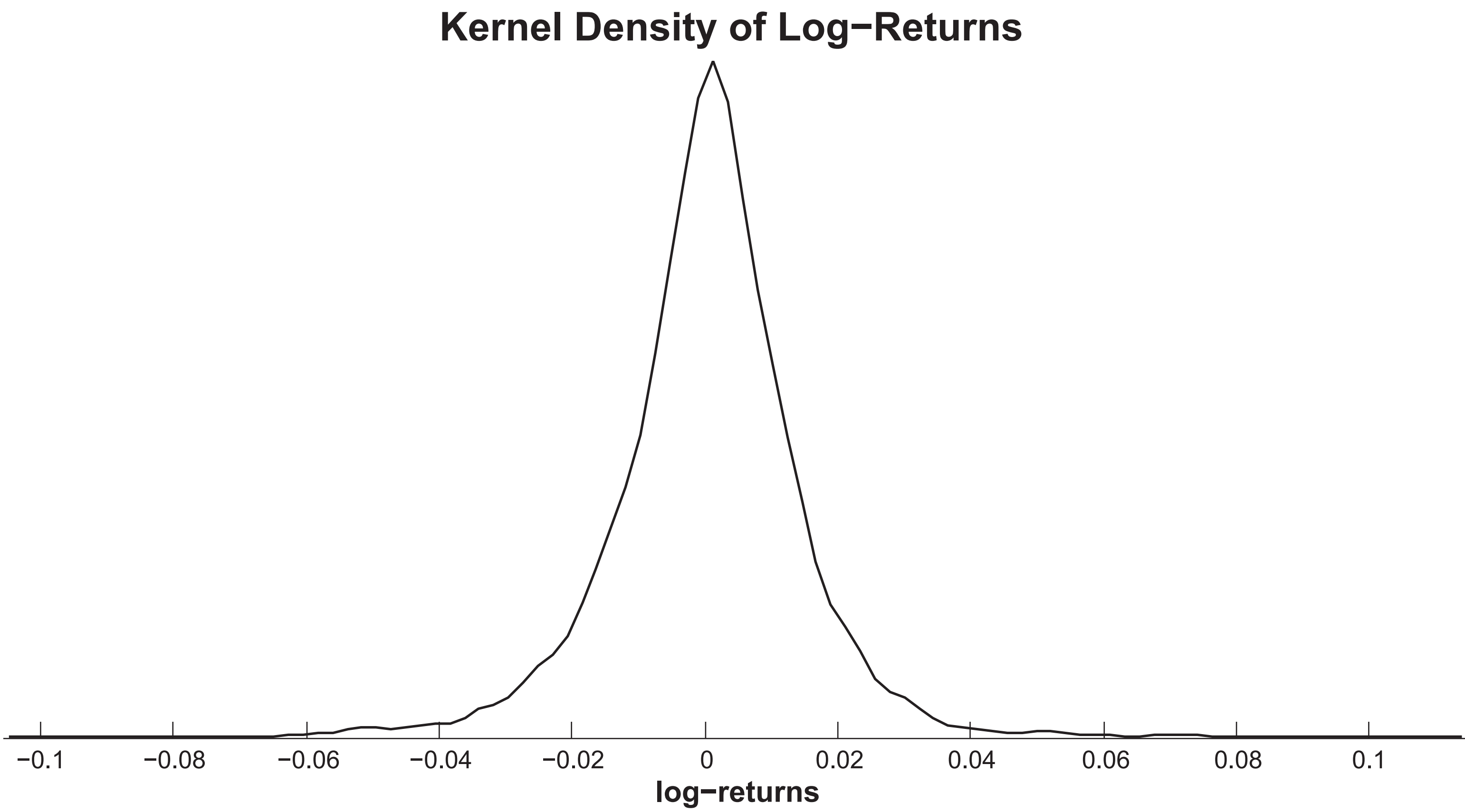
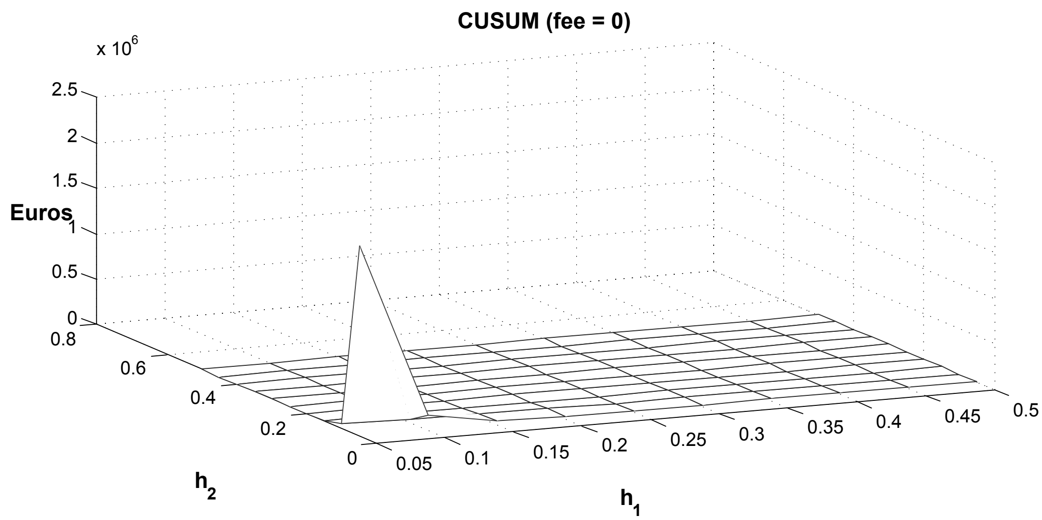
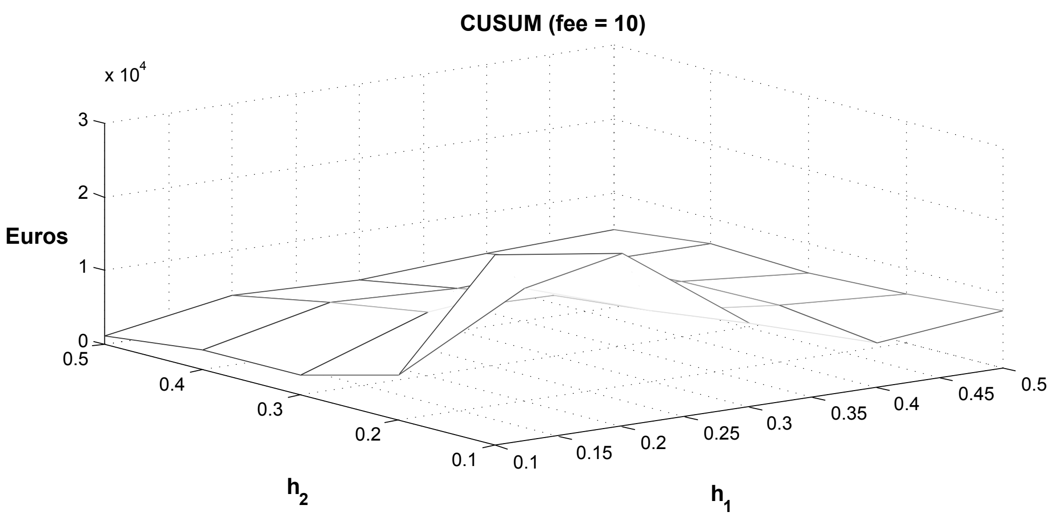


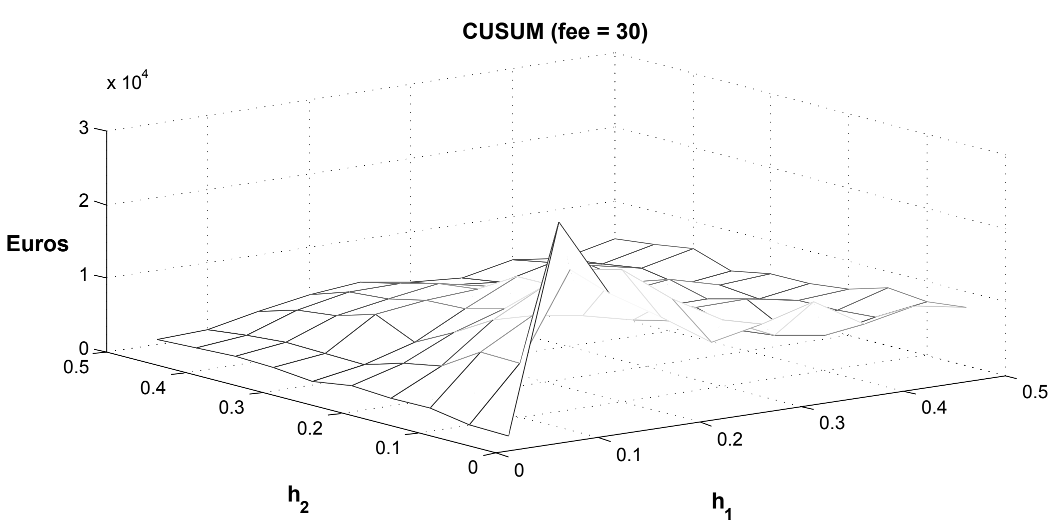
Figure 3: Kernel density estimation.



(a)

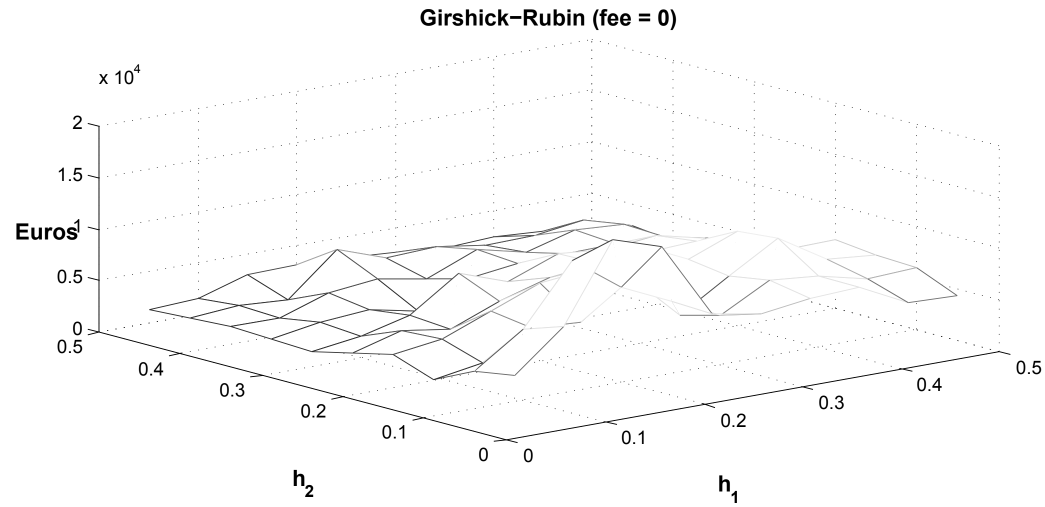


(b)

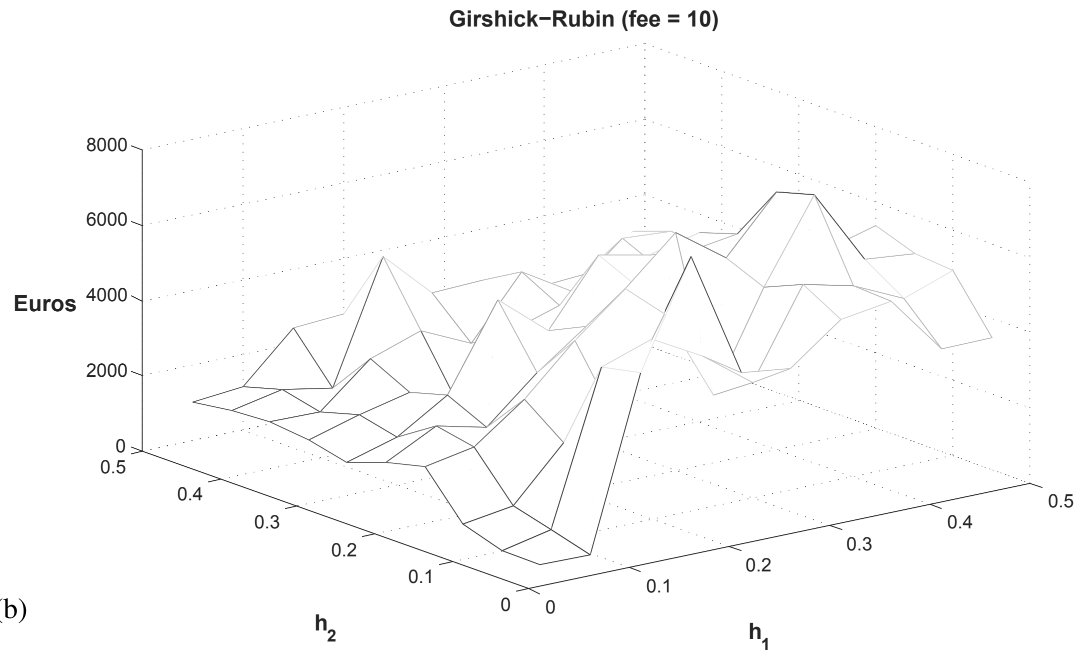


(c)

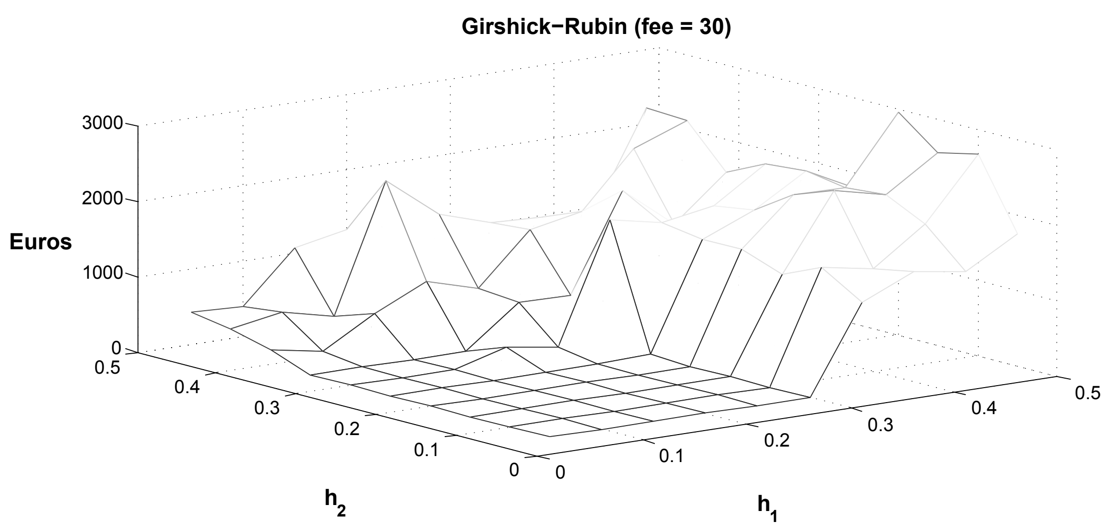
Figure 4: Comparison of CUSUM strategies for different fees ($h_1, h_2 \in \{0.05, 0.10, \ldots, 0.50\}$).



(a)



(b)



(c)

Figure 5: Comparison of Girshick-Rubin strategies for different fees ($h_1, h_2 \in \{0.05, 0.10, \ldots, 0.50\}$).

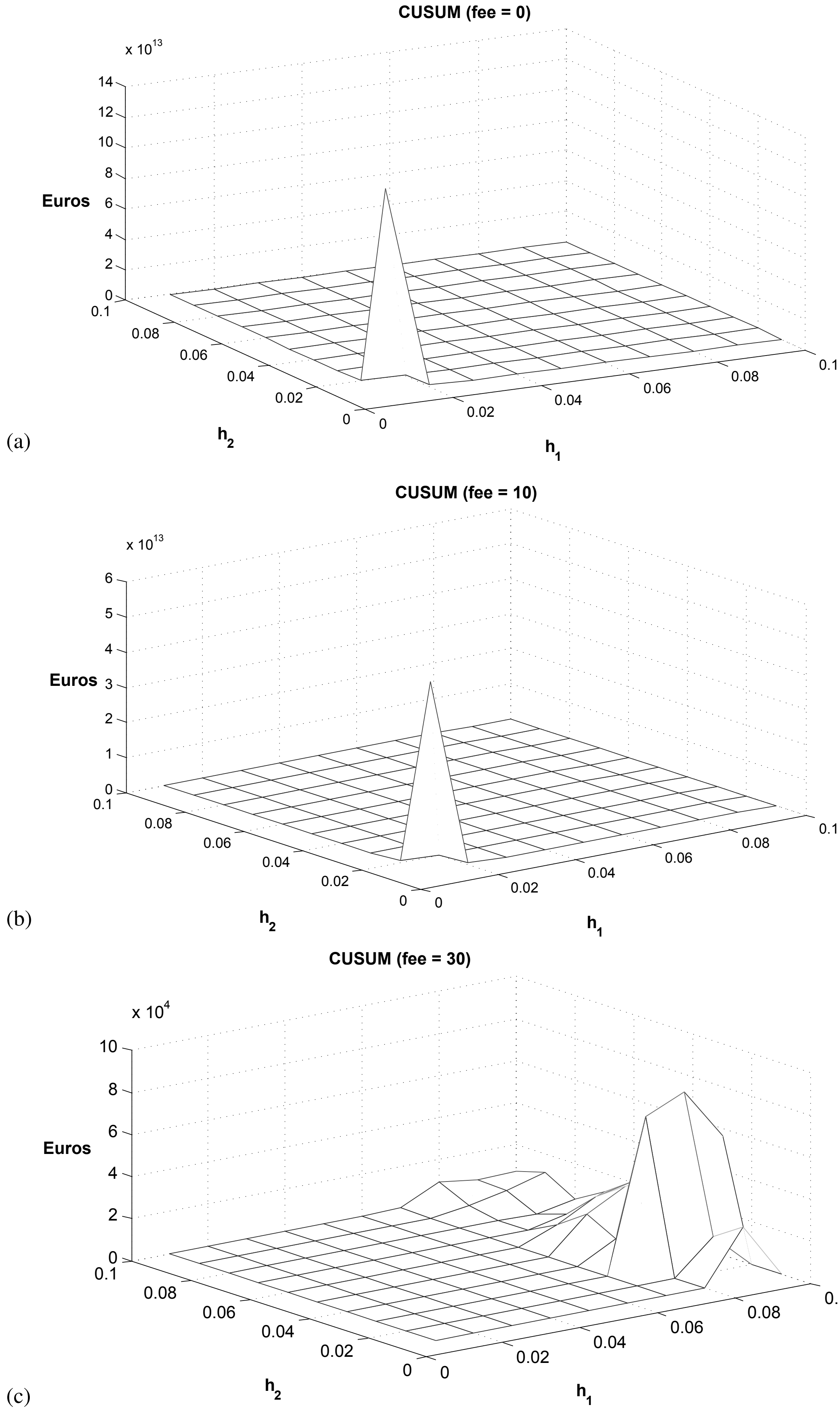
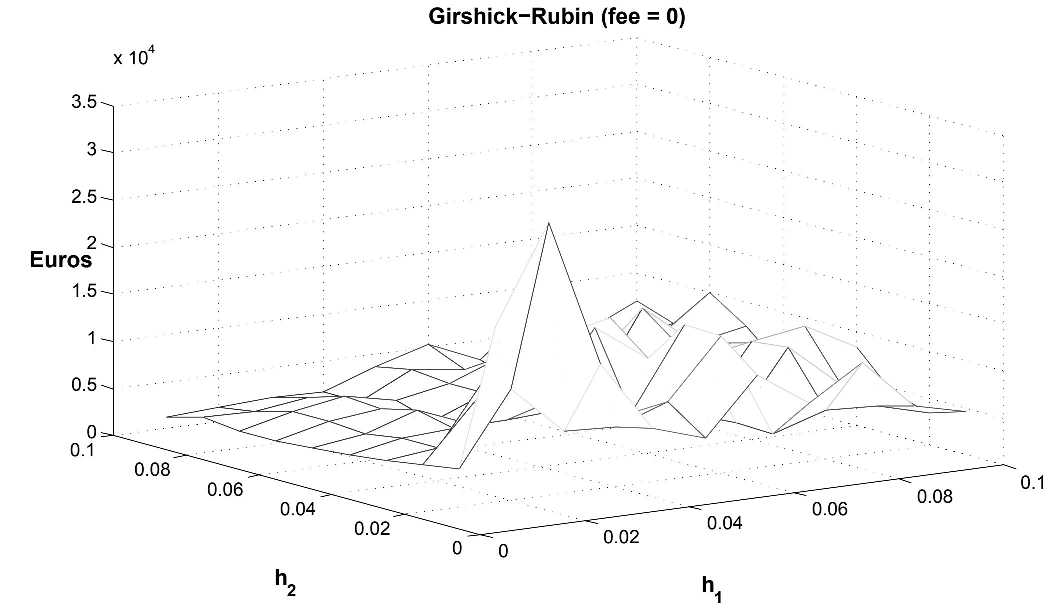
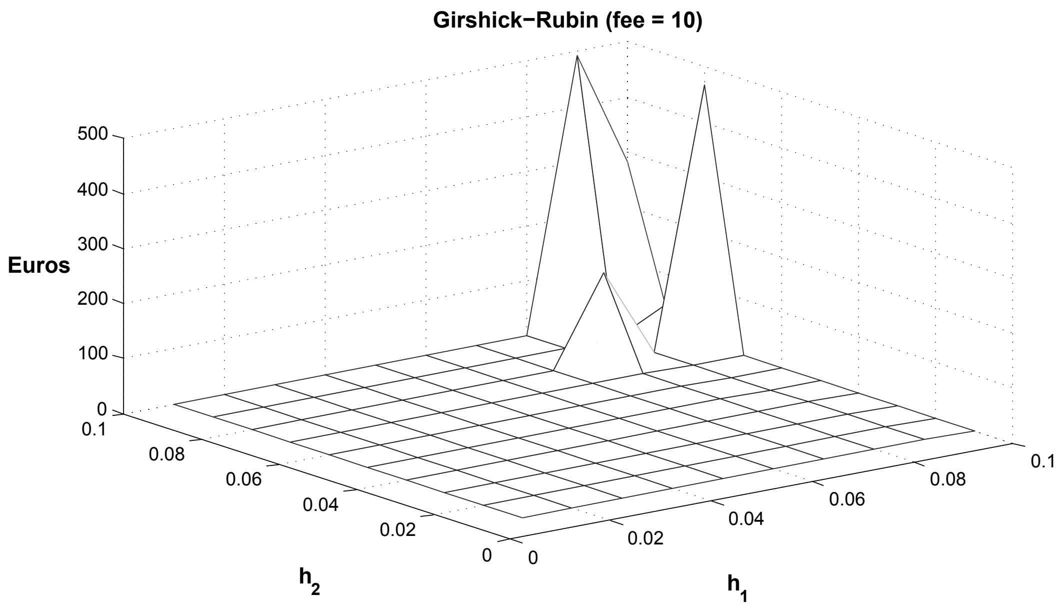


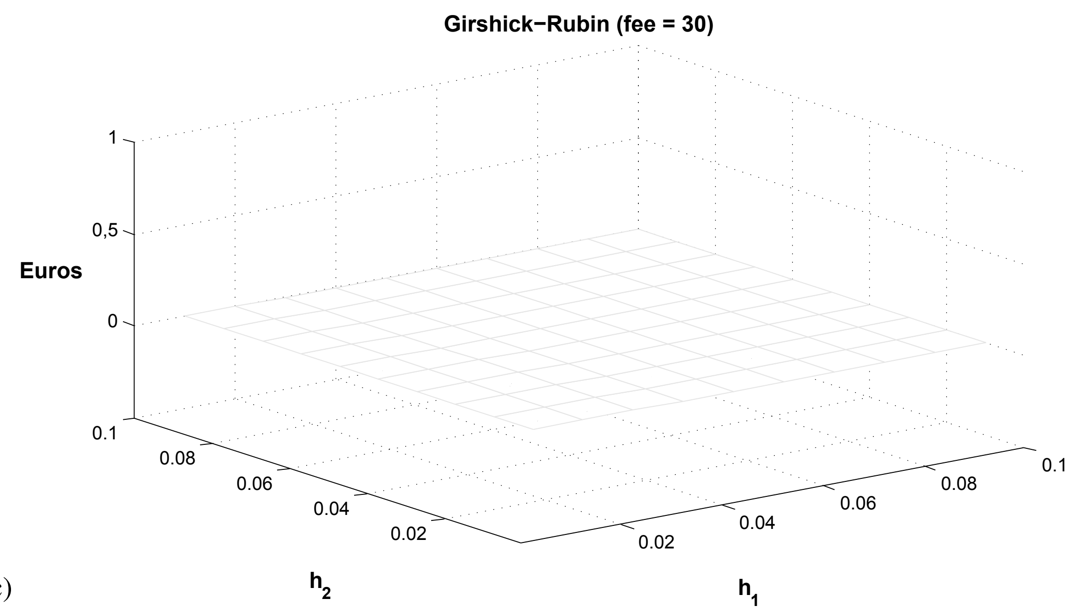
Figure 6: Comparison of CUSUM strategies for different fees ($h_1, h_2 \in \{0.01, 0.02, \ldots, 0.10\}$).



(a)



(b)



(c)

Figure 7: Comparison of Girshick-Rubin strategies for different fees ($h_1, h_2 \in \{0.01, 0.02, \ldots, 0.10\}$).

Grid		**{0.01, 0.02, ..., 0.10}**		
Fee	Strategy	Pay-off	h_1	h_2
0	CUSUM	133295220631634.11	0.01	0.01
	G-R	30508.40	0.01	0.02
10	CUSUM	54257469792643.15	0.01	0.01
	G-R	1000.00[1]		
30	CUSUM	80957.02	0.03	0.09
	G-R	1000.00[1]		

Grid		**{0.05, 0.10, ..., 0.50}**		
Fee	Strategy	Pay-off	h_1	h_2
0	CUSUM	2099809.36	0.05	0.05
	G-R	15866.21	0.05	0.15
10	CUSUM	1204424.63	0.10	0.10
	G-R	7331.32	0.05	0.20
30	CUSUM	27899.19	0.05	0.10
	G-R	2951.79	0.20	0.50

Grid		**{0.20, 0.21, ..., 0.40}**		
Fee	Strategy	Pay-off	h_1	h_2
0	G-R	12857.97	0.20	0.39
10	G-R	9703.03	0.20	0.39
30	G-R	2133.79	0.40	0.40

Table 1: Final optimal pay-off per strategy and grid.[1] The strategy leads to loss or ruin for any pair of threshold values.